Magnetism

Fifth lecture

Induced Electromotive Force Faraday's and Lenz's Law

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first stage

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1. Induction and Inductance

In fourth lecture we discussed the fact that a current produces a magnetic field. That fact came as a surprise to the scientists who discovered the effect. Perhaps even more surprising was the discovery of the reverse effect: A magnetic field can produce an electric field that can drive a current. This link between a magnetic field and the electric field it produces (induces) is now called Faraday's law of induction. The observations by Michael Faraday and other scientists that led to this law were at first just basic science. Today, however, applications of that basic science are almost everywhere. For example, induction is the basis of the electric guitars that revolutionized early rock and still drive heavy metal and punk today. It is also the basis of the electric generators that power cities and transportation lines and of the huge induction furnaces that are commonplace in foundries where large amounts of metal must be melted rapidly. Before we get to applications like the electric guitar, we must examine two simple experiments about Faraday's law of induction.

2. Two Experiments

Let us examine two simple experiments to prepare for our discussion of Faraday's law of induction.

First Experiment. Figure (1) shows a conducting loop connected to a sensitive ammeter. Because there is no battery or other source of electromotive force (emf) included, there is no current in the circuit. However, if we move a bar magnet toward the loop, a current suddenly appears in the circuit. The current disappears when the magnet stops. If we then move the magnet away, a current again suddenly appears, but now in the opposite direction. If we experimented for a while, we would discover the following:

1. A current appears only if there is relative motion between the loop and the magnet (one must move relative to the other); the current disappears when the relative motion between them ceases.

2. Faster motion produces a greater current.

3. If moving the magnet's north pole toward the loop causes, say, clockwise current, then moving the north pole away causes counterclockwise current. Moving the south pole toward or away from the loop also causes currents, but in the reversed directions.

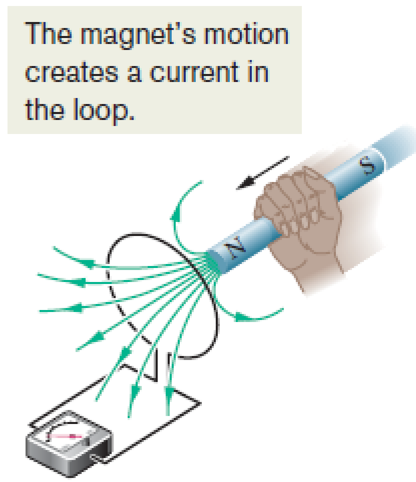

Figure (1): An ammeter registers a current in the wire loop when the magnet is moving with respect to the loop.

The current produced in the loop is called an **induced current**; the work done per unit charge to produce that current (to move the conduction electrons that constitute the current) is called an **induced emf**; and the process of producing the current and emf is called **induction**.

Second Experiment. For this experiment we use the apparatus of Figure (2), with the two conducting loops close to each other but not touching. If we close switch *S*, to turn on a current in the right-hand loop, the meter suddenly and briefly registers a current—an induced current—in the left-hand loop. If we then open the switch, another sudden and brief induced current appears in the left-hand loop, but in the opposite direction. We get an induced current (and thus an induced emf) only when the current in the right-hand loop is changing (either turning on or turning off) and not when it is constant (even if it is large).

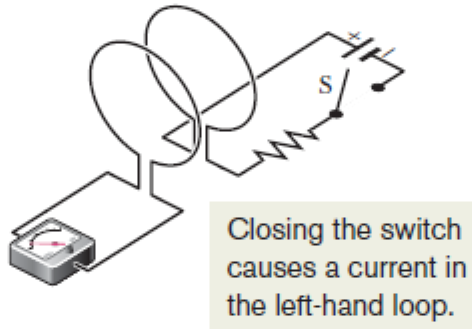

Figure (2): An ammeter registers a current in the left-hand wire loop just as switch S is closed (to turn on the current in the right hand wire loop) or opened (to turn off the current in the right-hand loop). No motion of the coils is involved.

The induced emf and induced current in these experiments are apparently caused when something changes—but what is that "something"? Faraday knew.

3. Faraday's Law of Induction

Faraday realized that an emf and a current can be induced in a loop, as in our two experiments, by changing the *amount of magnetic field* passing through the loop. He further realized that the "amount of magnetic field" can be visualized in terms of the magnetic field lines passing through the loop. **Faraday's law of induction,** stated in terms of our experiments, is this:

An emf is induced in the loop at the left in Figure (1&2) when the number of magnetic field lines that pass through the loop is changing.

The actual number of field lines passing through the loop does not matter; the values of the induced emf and induced current are determined by the *rate* at which that number changes.

In our first experiment Figure (1), the magnetic field lines spread out from the north pole of the magnet. Thus, as we move the north pole closer to the loop, the number of field lines passing through the loop increases. That increase apparently causes conduction electrons in the loop to move (the induced current) and provides energy (the induced emf) for their motion. When the magnet stops moving, the number of field lines through the loop no longer changes and the induced current and induced emf disappear.

In our second experiment Figure (2), when the switch is open (no current), there are no field lines. However, when we turn on the current in the right hand loop, the increasing current builds up a magnetic field around that loop and at the left-hand loop. While the field builds, the number of magnetic field lines through the left-hand loop increases. As in the first experiment, the increase in field lines through that loop apparently induces a current and an emf there. When the current in the right-hand loop reaches a final, steady value, the number of field lines through the left-hand loop no longer changes, and the induced current and induced emf disappear.

A Quantitative Treatment

To put Faraday's law to work, we need a way to calculate the *amount of magnetic field* that passes through a loop. In a similar situation, we needed to calculate the amount of electric field that passes through a surface. There we defined an electric flux $\Phi_E = \int \vec{E}.d\vec{A}$. Here we define a *magnetic flux:* Suppose a loop enclosing an area A is placed in a magnetic field $\vec{B}$. Then the **magnetic flux** through the loop is:

$$\Phi_B = \int \vec{B}.d\vec{A} \quad (magnetic\ flux\ through\ area\ A) \qquad (1)$$

As in previous chapter, $d\vec{A}$ is a vector of magnitude dA that is perpendicular to a differential area dA. As with electric flux, we want the component of the field that pierces the surface (not skims along it). The dot product of the field and the area vector automatically gives us that piercing component.

Special Case. As a special case of Equation (1), suppose that the loop lies in a plane and that the magnetic field is perpendicular to the plane of the loop. Then we can write the dot product in Equation (1) as $B\ dA \cos 0° = B\ dA$. If the magnetic field is also uniform, then B can be brought out in front of the integral sign. The remaining $\int dA$ then gives just the area A of the loop. Thus, Equation (1) reduces to:

$$\Phi_B = BA \ \left(\vec{B} \perp area\ A, \vec{B}\ uniform\right) \qquad (2)$$

Unit. From Equation (1) and (2), we see that the SI unit for magnetic flux is the tesla–square meter, which is called the weber (abbreviated Wb):

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T.m}^2 \qquad (3)$$

Faraday's Law. With the notion of magnetic flux, we can state Faraday's law in a more quantitative and useful way:

The magnitude of the emf ζ induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.

As you will see below, the induced emf ζ tends to oppose the flux change, so Faraday's law is formally written as:

$$\zeta = -\frac{d\Phi_B}{dt} \ (Faraday's law) \qquad (4)$$

with the minus sign indicating that opposition. We often neglect the minus sign in Equation (4), seeking only the magnitude of the induced emf.

If we change the magnetic flux through a coil of N turns, an induced emf appears in every turn and the total emf induced in the coil is the sum of these individual induced emfs. If the coil is tightly wound (*closely packed*), so that the same magnetic flux Φ_B passes through all the turns, the total emf induced in the coil is:

$$\zeta = -N\frac{d\Phi_B}{dt} \ (coil\ of\ N\ turns) \qquad (5)$$

Here are the general means by which we can change the magnetic flux through a coil:

1. Change the magnitude B of the magnetic field within the coil.

2. Change either the total area of the coil or the portion of that area that lies within the magnetic field (for example, by expanding the coil or sliding it into or out of the field).

3.Change the angle between the direction of the magnetic field $\vec{B}$ and the plane of the coil (for example, by rotating the coil so that field $\vec{B}$ is first perpendicular to the plane of the coil and then is along that plane).

Example:

The long solenoid S shown (in cross section) in Fig. 30-3 has 220 turns/cm and carries a current $i = 1.5$ A; its diameter D is 3.2 cm. At its center we place a 130-turn closely packed coil C of diameter $d = 2.1$ cm. The current in the solenoid is reduced to zero at a steady rate in 25 ms. What is the magnitude of the emf that is induced in coil C while the current in the solenoid is changing?

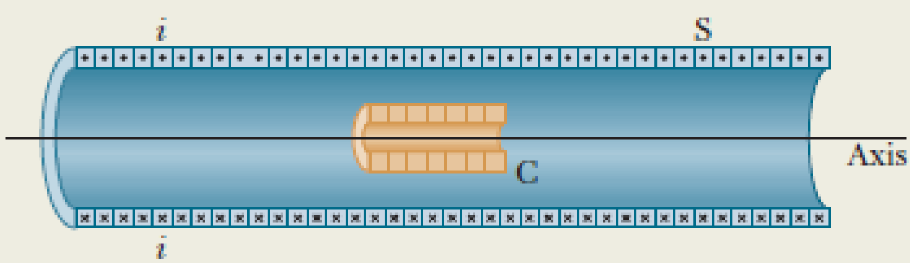

Figure 30-3 A coil C is located inside a solenoid S, which carries current i.

Calculations: Because coil C consists of more than one turn, we apply Faraday's law in the form of Eq. 30-5 ($\mathscr{E} = -N\, d\Phi_B/dt$), where the number of turns N is 130 and $d\Phi_B/dt$ is the rate at which the flux changes.

Because the current in the solenoid decreases at a steady rate, flux Φ_B also decreases at a steady rate, and so we can write $d\Phi_B/dt$ as $\Delta\Phi_B/\Delta t$. Then, to evaluate $\Delta\Phi_B$, we need the final and initial flux values. The final flux $\Phi_{B,f}$ is zero because the final current in the solenoid is zero. To find the initial flux $\Phi_{B,i}$, we note that area A is $\frac{1}{4}\pi d^2$ ($= 3.464 \times 10^{-4}\ \text{m}^2$) and the number n is 220 turns/cm, or 22 000 turns/m. Substituting Eq. 29-23 into Eq. 30-2 then leads to

$$\begin{aligned}\Phi_{B,i} &= BA = (\mu_0 in)A \\ &= (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1.5\ \text{A})(22\,000\ \text{turns/m}) \\ &\quad \times (3.464 \times 10^{-4}\ \text{m}^2) \\ &= 1.44 \times 10^{-5}\ \text{Wb}.\end{aligned}$$

Now we can write

$$\begin{aligned}\frac{d\Phi_B}{dt} = \frac{\Delta\Phi_B}{\Delta t} &= \frac{\Phi_{B,f} - \Phi_{B,i}}{\Delta t} \\ &= \frac{(0 - 1.44 \times 10^{-5}\ \text{Wb})}{25 \times 10^{-3}\ \text{s}} \\ &= -5.76 \times 10^{-4}\ \text{Wb/s} \\ &= -5.76 \times 10^{-4}\ \text{V}.\end{aligned}$$

We are interested only in magnitudes; so we ignore the minus signs here and in Eq. 30-5, writing

$$\begin{aligned}\mathscr{E} = N\frac{d\Phi_B}{dt} &= (130\ \text{turns})(5.76 \times 10^{-4}\ \text{V}) \\ &= 7.5 \times 10^{-2}\ \text{V} \\ &= 75\ \text{mV}. \qquad \text{(Answer)}\end{aligned}$$

4. Lenz's Law

Soon after Faraday proposed his law of induction, Heinrich Friedrich Lenz devised a rule for determining the direction of an induced current in a loop:

An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current.

Furthermore, the direction of an induced emf is that of the induced current. The key word in Lenz's law is "opposition." Let's apply the law to the motion of the north pole toward the conducting loop in Figure (3).

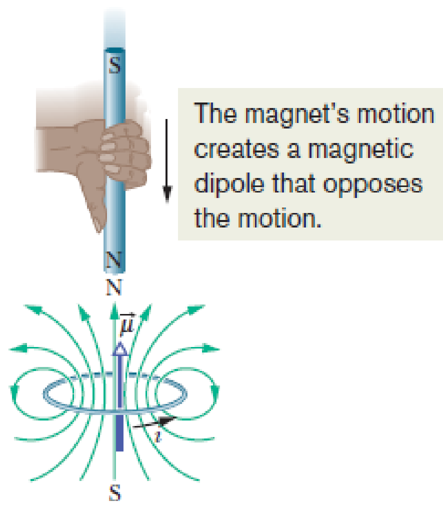

Figure (3): Lenz's law at work. As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment oriented so as to oppose the motion of the magnet. Thus, the induced current must be counterclockwise as shown.

1. Opposition to Pole Movement. The approach of the magnet's north pole in Figure (3) increases the magnetic flux through the loop and thereby induces a current in the loop. From Figure (4), we know that the loop then acts as a magnetic dipole with a south pole and a north pole, and that its magnetic dipole moment $\vec{\mu}$ is directed from south to north. To oppose the magnetic flux increase being caused by the approaching magnet, the loop's north pole (and thus $\vec{\mu}$) must face toward the approaching north pole so as to repel it Figure (3). Then the curled – straight right-hand rule for $\vec{\mu}$ (Figure (4)) tells us that the current induced in the loop must be counterclockwise in Figure (3).

If we next pull the magnet away from the loop, a current will again be induced in the loop. Now, however, the loop will have a south pole facing the retreating north pole of the magnet, so as to oppose the retreat. Thus, the induced current will be clockwise.

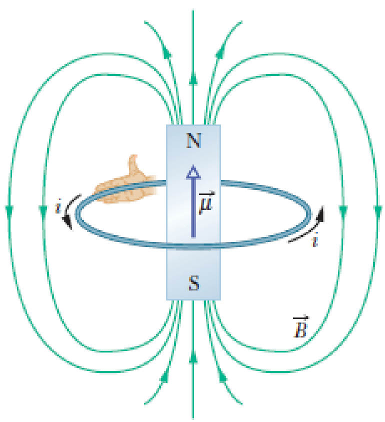

Figure (4): A current loop produces a magnetic field like that of a bar magnet and thus has associated north and south poles. The magnetic dipole moment of the loop, its direction given by a curled–straight right-hand rule, points from the south pole to the north pole, in the direction of the field *B* within the loop.

2. Opposition to Flux Change. In Figure (3), with the magnet initially distant, no magnetic flux passes through the loop. As the north pole of the magnet then nears the loop with its magnetic field $\vec{B}$ directed downward, the flux through the loop increases. To oppose this increase in flux, the induced current i must set up its own field $\vec{B}_{ind}$ directed upward inside the loop, as shown in Figure (5-a); then the upward flux of field $\vec{B}_{ind}$ opposes the increasing downward flux of field $\vec{B}$. The curled–straight right-hand rule of Figure (4) then tells us that I must be counterclockwise in Figure (5-a).

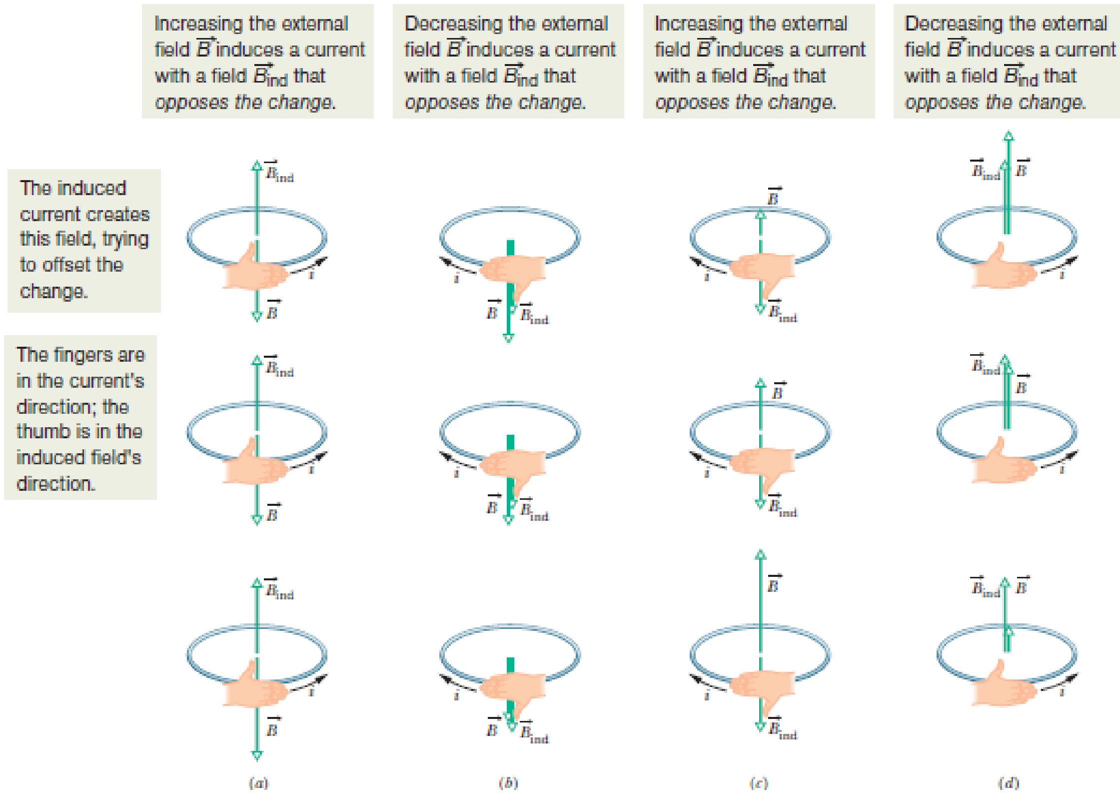

Figure (5): The direction of the current i induced in a loop is such that the current's magnetic field $\vec{B}_{ind}$ opposes the *change* in the magnetic field $\vec{B}$ inducing i. The field $\vec{B}_{ind}$ is always directed opposite an increasing field $\vec{B}(a,c)$ and in the same direction as a decreasing field $\vec{B}(b,d)$. The curled–straight right-hand rule gives the direction of the induced current based on the direction of the induced field.

Heads Up. The flux of $\vec{B}_{ind}$ always opposes the change in the flux of $\vec{B}$, but $\vec{B}_{ind}$ is not always opposite . For example, if we next pull the magnet away from the loop in Figure (3), the magnet's flux Φ_B is still downward through the loop, but it is now decreasing. The flux of $\vec{B}_{ind}$ must now be downward inside the loop, to oppose that decrease Figure (5-b). Thus, $\vec{B}_{ind}$ and $\vec{B}$ are now in the same direction. In Figure (5-c & d), the south pole of the magnet approaches and retreats from the loop, again with opposition to change.

Figure 30-6 shows a conducting loop consisting of a half-circle of radius $r = 0.20$ m and three straight sections. The half-circle lies in a uniform magnetic field $\vec{B}$ that is directed out of the page; the field magnitude is given by $B = 4.0t^2 + 2.0t + 3.0$, with B in teslas and t in seconds. An ideal battery with emf $\mathscr{E}_{bat} = 2.0$ V is connected to the loop. The resistance of the loop is 2.0 Ω.

(a) What are the magnitude and direction of the emf $\mathscr{E}_{ind}$ induced around the loop by field $\vec{B}$ at $t = 10$ s?

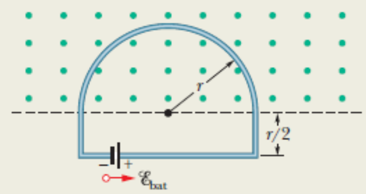

Figure 30-6 A battery is connected to a conducting loop that includes a half-circle of radius r lying in a uniform magnetic field. The field is directed out of the page; its magnitude is changing.

KEY IDEAS

Magnitude: Using Eq. 30-2 and realizing that only the field magnitude B changes in time (not the area A), we rewrite Faraday's law, Eq. 30-4, as

$$\mathscr{E}_{ind} = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt}.$$

Because the flux penetrates the loop only within the half-circle, the area A in this equation is $\frac{1}{2}\pi r^2$. Substituting this and the given expression for B yields

$$\mathscr{E}_{ind} = A\frac{dB}{dt} = \frac{\pi r^2}{2}\frac{d}{dt}(4.0t^2 + 2.0t + 3.0)$$

$$= \frac{\pi r^2}{2}(8.0t + 2.0).$$

At $t = 10$ s, then,

$$\mathscr{E}_{ind} = \frac{\pi(0.20\text{ m})^2}{2}[8.0(10) + 2.0]$$

$$= 5.152\text{ V} \approx 5.2\text{ V}. \qquad \text{(Answer)}$$

Direction: To find the direction of $\mathscr{E}_{ind}$, we first note that in Fig. 30-6 the flux through the loop is out of the page and increasing. Because the induced field B_{ind} (due to the induced current) must oppose that increase, it must be *into* the page. Using the curled–straight right-hand rule (Fig. 30-5*c*), we find that the induced current is clockwise around the loop, and thus so is the induced emf $\mathscr{E}_{ind}$.

(b) What is the current in the loop at $t = 10$ s?