Al-Mustaqbal University College
Department of Computer
Engineering Techniques

Information Theory and coding

Fourth stage

Lecture 4

Self- information

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Model of information transmission system

Transmitting a message from a transmitter to a receiver can be sketched as in Figure 1:

The components of information system as described by Shannon are:

1- An information source is a device which randomly delivers symbols from an alphabet. As an example, a PC (Personal Computer) connected to internet is an information source which produces binary digits from the binary alphabet {0, 1}.
2- A source encoder allows one to represent the data source more compactly by eliminating redundancy: it aims to reduce the data rate.
3- A channel encoder adds redundancy to protect the transmitted signal against transmission errors.

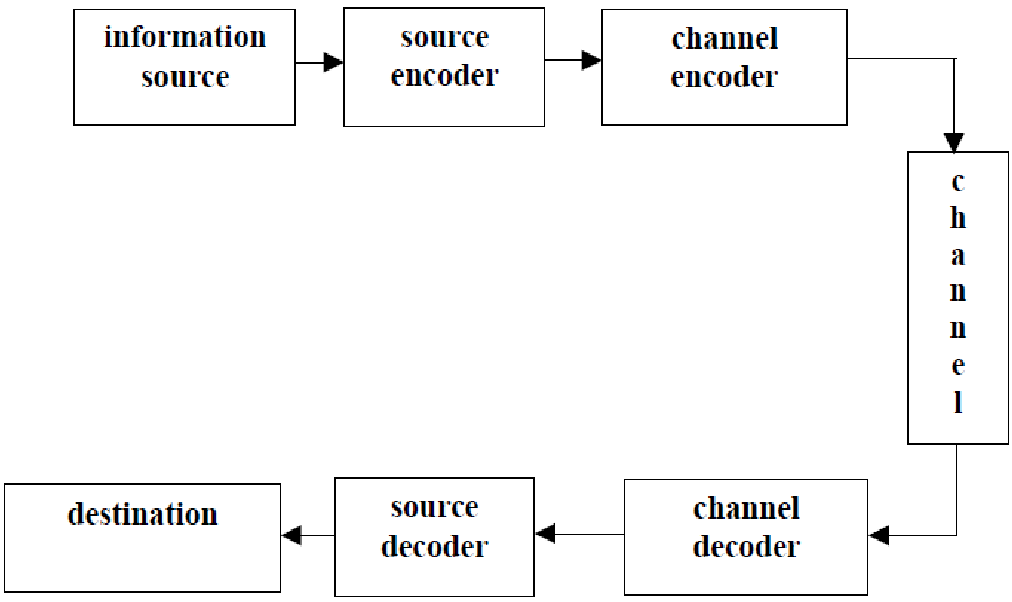

Figure 1: Shannon paradigm

4- A channel is a system which links a transmitter to a receiver. It includes signaling equipment and pair of copper wires or coaxial cable or optical fiber, among other possibilities.
5- The rest of blocks is the receiver end; each block has inverse processing to the corresponding transmitted end.

Self- information:

In information theory, **self-information** is a measure of the information content associated with the *outcome* of a random variable. It is expressed in a unit of information, for example bits, nats, or hartleys, depending on the base of the logarithm used in its calculation.

A **bit** is the basic unit of information in computing and digital communications. A bit can have only one of two values, and may therefore be physically implemented with a two-state device. These values are most commonly represented as 0 and 1.

A **nat** is the **natural unit of information**, sometimes also **nit** or **nepit**, is a unit of information or entropy, based on natural logarithms and powers of *e*, rather than the powers of 2 and base 2 logarithms which define the bit. This unit is also known by its unit symbol, the nat.

The **hartley** (symbol **Hart**) is a unit of information defined by International Standard IEC 80000-13 of the International Electrotechnical Commission. One hartley is the information content of an event if the probability of that event occurring is 1/10. It is therefore equal to the information contained in one decimal digit (or dit).

1 Hart ≈ 3.322 Sh ≈ 2.303 nat.

The amount of self-information contained in a probabilistic event depends only on the probability of that event: the smaller its probability, the larger the self-information associated with receiving the information that the event indeed occurred as shown in Figure 2.

i- Information is zero if $p(x_i) = 1$ (certain event)

ii- Information increase as $p(x_i)$ decrease to zero

iii- Information is a +ve quantity

The log function satisfies all previous three points hence:

$$I(x_i) = log_a \frac{1}{p(x_i)} = -log_a p(x_i)$$

Where $I(x_i)$ is self-information of (x_i) and if:

i- If “a” =2 , then $I(x_i)$ has the unit of bits

ii- If “a”= e = 2.71828, then $I(x_i)$ has the unit of nats

iii- If “a”= 10, then $I(x_i)$ has the unit of hartly

Note: $\log_a x = \frac{ln\ x}{ln\ a}$

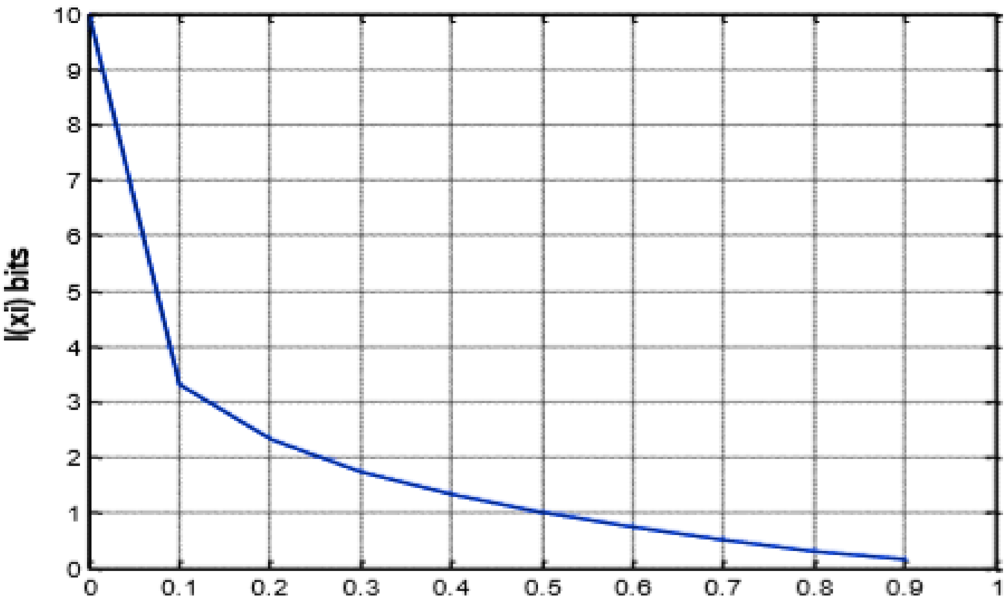

Figure 2: Relation between probability and self-information

Example 1: A fair die is thrown, find the amount of information gained if you are told that 4 will appear.

Solution:

$$P(1) = P(2) = \cdots \dots \dots . = P(6) = \frac{1}{6}$$

Then:

$$I(4) = -log_2\left(\frac{1}{6}\right) = -\frac{\ln(\frac{1}{6})}{\ln 2} = 2.5849\ \boldsymbol{bits}$$

Or

$$I(4) = -log_e\left(\frac{1}{6}\right) = -\frac{\ln(\frac{1}{6})}{\ln e} = 1.791 \quad \boldsymbol{nats}$$

Or

$$I(4) = -log_{10}\left(\frac{1}{6}\right) = -\frac{\ln(\frac{1}{6})}{\ln 10} = 0.778 \quad \boldsymbol{hart}$$

Example 2: A biased coin has P(Head)=0.3. Find the amount of information gained if you are told that a tail will appear.

Solution:

$$P(tail) = 1 - P(Head) = 1 - 0.3 = 0.7$$

$$I(tail) = -log_2(0.7) = -\frac{\ln(0.7)}{\ln 2} = 0.5145\ bits$$

HW: A communication system source emits the following information with their corresponding probabilities as follows: A=1/2, B=1/4, C=1/8. Calculate the information conveyed by each source outputs. ***Draw the relation between probability and self-information.***