Magnetism

Sixth lecture

Induction types and RL Circuits

Dr. Mohammed Hashim Abbas

first stage

Department of medical physics

Al-Mustaqbal University-College

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1. Self-Induction

If two coils—which we can now call inductors—are near each other, a current *i* in one coil produces a magnetic flux Φ_B through the second coil. We have seen that if we change this flux by changing the current, an induced emf appears in the second coil according to Faraday's law. An induced emf appears in the first coil as well.

An induced emf ζ_L appears in any coil in which the current is changing.

This process (see Figure (1)) is called **self-induction,** and the emf that appears is called a **self-induced emf.** It obeys Faraday's law of induction just as other induced emfs do.

The **inductance *L*** of the inductor is then defined in terms of that magnetic flux Φ_B as:

$$L = \frac{N\Phi_B}{i} \quad (1)$$

in which *N* is the number of turns and *i* is a current.

For any inductor, Equation (1) tells us that:

$$N\Phi_B = Li \quad (2)$$

Faraday's law tells us that:

$$\zeta_L = -\frac{d(N\Phi_B)}{dt} \quad (3)$$

By combining equations (2) and (3) we can write:

$$\zeta_L = -L\,\frac{di}{dt}\ \ (self - induced\ emf) \quad (4)$$

Thus, in any inductor (such as a coil, a solenoid, or a toroid) a self-induced emf appears whenever the current changes with time. The magnitude of the current has no influence on the magnitude of the induced emf, only the rate of change of the current counts.

Direction. You can find the *direction* of a self-induced emf from Lenz's law. The minus sign in Equation (4) indicates that—as the law states—the self-induced emf ζ_L has the orientation such that it opposes the change in current i. We can drop the minus sign when we want only the magnitude of ζ_L.

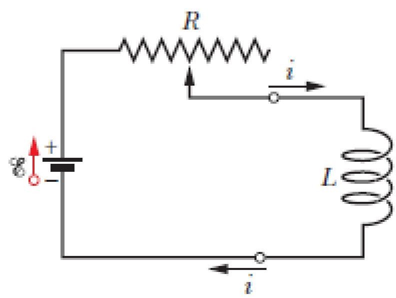

Figure (1): If the current in a coil is changed by varying the contact position on a variable resistor, a self-induced emf ζ_L. will appear in the coil *while the current is changing*.

2. RL Circuits

We saw that if we suddenly introduce an emf ζ into a single-loop circuit containing a resistor R and a capacitor C, the charge on the capacitor does not build up immediately to its final equilibrium value $C\zeta$ but approaches it in an exponential fashion:

$$q = C\mathscr{E}(1 - e^{-t/\tau_C}). \qquad (5)$$

The rate at which the charge builds up is determined by the capacitive time constant τ_C, defined as:

$$\tau_C = RC.$$

If we suddenly remove the emf from this same circuit, the charge does not immediately fall to zero but approaches zero in an exponential fashion:

$$q = q_0 e^{-t/\tau_C}. \qquad (6)$$

The time constant τ_C, describes the fall of the charge as well as its rise.

An analogous slowing of the rise (or fall) of the current occurs if we introduce an emf ζ into (or remove it from) a single-loop circuit containing a resistor R and an inductor L.

When the switch S in Figure (2) is closed on a, for example, the current in the resistor starts to rise. If the inductor were not present, the current would rise rapidly to a steady value ζ/R. Because of the inductor, however, a self-induced emf ζ_L appears in the circuit; from Lenz's law, this emf opposes the rise of the current, which means that it opposes the battery emf ζ in polarity. Thus, the current in the resistor responds to the difference between two emfs, a constant ζ due to the battery and a variable ζ_L (= $-L\, di/dt$) due to self-induction. As long as this ζ_L is present, the current will be less than ζ/R.

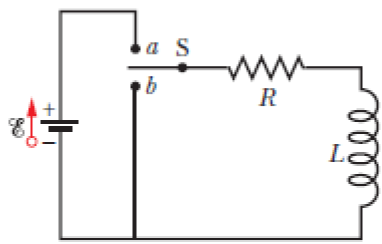

Figure (2) An *RL* circuit. When switch S is closed on *a*, the current rises and approaches a limiting value ζ/R.

Initially, an inductor acts to oppose changes in the current through it. A long time later, it acts like ordinary connecting wire.

Now let us analyze the situation quantitatively. With the switch S in Figure (2) thrown to a, the circuit is equivalent to that of Figure (3). Let us apply the loop rule, starting at point x in this figure and moving clockwise around the loop along with current i.

1. *Resistor*. Because we move through the resistor in the direction of current i, the electric potential decreases by iR. Thus, as we move from point x to point y, we encounter a potential change of $-iR$.
2. *Inductor*. Because current i is changing, there is a self-induced emf ζ_L in the inductor. The magnitude of ζ_L is given by Equation (4) as $L\, di/dt$. The direction of ζ_L is upward in Figure (3) because current i is downward through the inductor *and* increasing. Thus, as we move from point y to point z, opposite the direction of ζ_L, we encounter a potential change of $-L\, di/dt$.

3. *Battery*. As we move from point z back to starting point x, we encounter a potential change of $+\zeta$ due to the battery's emf.

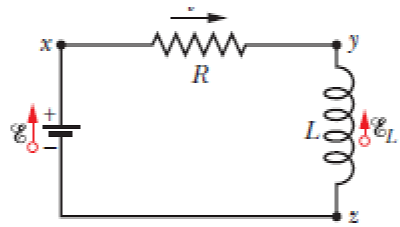

Figure (3) The circuit of Fig. 30-15 with the switch closed on *a*. We apply the loop rule for the circuit clockwise, starting at *x*.

Thus, the loop rule gives us:

$$L\frac{di}{dt} + Ri = \mathscr{E} \quad \text{(RL circuit).} \tag{7}$$

Equation (7) is a differential equation involving the variable i and its first derivative di/dt. To solve it, we seek the function $i(t)$ such that when $i(t)$ and its first derivative are substituted in Equation (7), the equation is satisfied and the initial condition $i(0) = 0$ is satisfied.

$$i = \frac{\mathscr{E}}{R}\left(1 - e^{-t/\tau_L}\right) \quad \text{(rise of current).} \tag{8}$$

Here τ_L, , the **inductive time constant,** is given by:

$$\tau_L = \frac{L}{R} \quad \text{(time constant).}$$

Let's examine Equation (7) for just after the switch is closed (at time $t = 0$) and for a time long after the switch is closed $t \rightarrow \infty$.

3. Mutual Induction

In this section we return to the case of two interacting coils, which we first discussed, and we treat it in a somewhat more formal manner. We saw earlier that if two coils are close together, a steady current i in one coil will set up a magnetic flux Φ through the other coil (*linking* the other coil). If we change i with time, an emf ζ given by Faraday's law appears in the second coil; we called this process *induction*. We could better have called it **mutual**

induction, to suggest the mutual interaction of the two coils and to distinguish it from *self-induction*, in which only one coil is involved.

Let us look a little more quantitatively at mutual induction. Figure (4-a) shows two circular close-packed coils near each other and sharing a common central axis. With the variable resistor set at a particular resistance R, the battery produces a steady current i_1 in coil 1. This current creates a magnetic field represented by the lines of B_1 in the figure. Coil 2 is connected to a sensitive meter but contains no battery; a magnetic flux Φ_{21} (the flux through coil 2 associated with the current in coil 1) links the N_2 turns of coil 2.

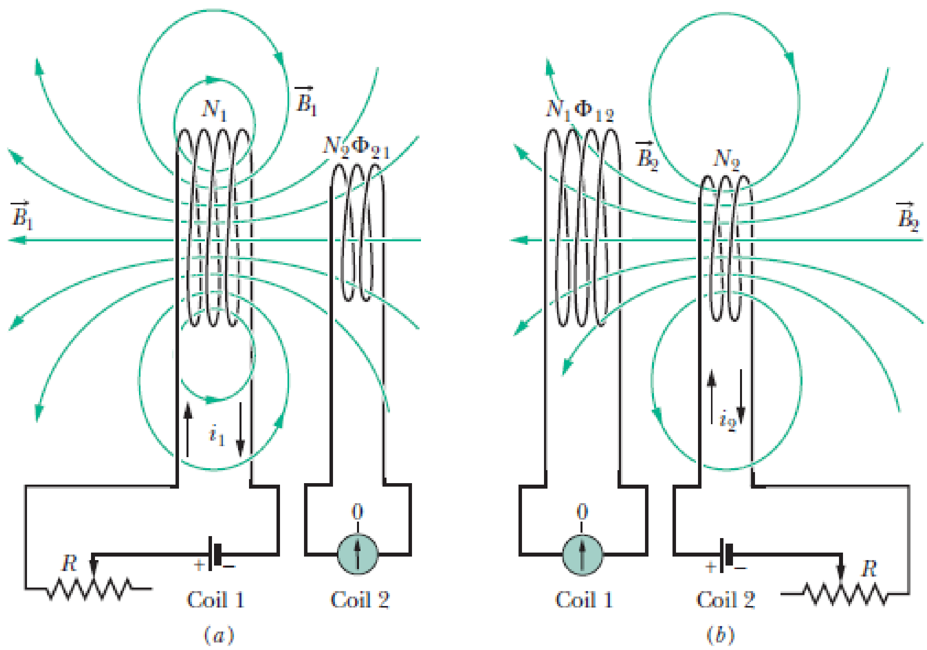

Figure (4) Mutual induction. (*a*) The magnetic field $\vec{B}_1$ produced by current i_1 in coil 1 extends through coil 2. If i_1 is varied (by varying resistance *R*), an emf is induced in coil 2 and current registers on the meter connected to coil 2. (*b*) The roles of the coils interchanged.

We define the mutual inductance M_{21} of coil 2 with respect to coil 1 as:

$$M_{21} = \frac{N_2\Phi_{21}}{i_1}, \tag{9}$$

which has the same form as Equation (2):

$$L = N\Phi/i,$$

the definition of inductance. We can recast Equation (9) as:

$$M_{21} i_1 = N_2 \Phi_{21}.$$

If we cause i_1 to vary with time by varying R, we have

$$M_{21} \frac{di_1}{dt} = N_2 \frac{d\Phi_{21}}{dt}.$$

The right side of this equation is, according to Faraday's law, just the magnitude of the emf ζ_2 appearing in coil 2 due to the changing current in coil 1.Thus, with a minus sign to indicate direction:

$$\mathscr{E}_2 = -M_{21} \frac{di_1}{dt},$$

Interchange. Let us now interchange the roles of coils 1 and 2, as in Figure (4-b); that is, we set up a current i_2 in coil 2 by means of a battery, and this produces a magnetic flux Φ_{12} that links coil 1. If we change i_2 with time by varying R, we then have, by the argument given above:

$$\mathscr{E}_1 = -M_{12} \frac{di_2}{dt}.$$

Thus, we see that the emf induced in either coil is proportional to the rate of change of current in the other coil. The proportionality constants M_{21} and $M12$ seem to be different. However, they turn out to be the same, although we cannot prove that fact here. Thus, we have:

$$M_{21} = M_{12} = M,$$

$$\mathscr{E}_2 = -M \frac{di_1}{dt}$$

$$\mathscr{E}_1 = -M \frac{di_2}{dt}.$$