Magnetism

Seventh lecture

Magnetism and Electrons

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1. MAGNETISM AND ELECTRONS

Magnetic materials, from lodestones to tattoos, are magnetic because of the electrons within them. We have already seen one way in which electrons can generate a magnetic field: Send them through a wire as an electric current, and their motion produces a magnetic field around the wire. There are two more ways, each involving a magnetic dipole moment that produces a magnetic field in the surrounding space. However, their explanation requires quantum physics that is beyond the physics presented in this book, and so here we shall only outline the results.

2. Spin Magnetic Dipole Moment

An electron has an intrinsic angular momentum called its **spin angular momentum** (or just **spin**) $\vec{S}$; associated with this spin is an intrinsic **spin magnetic dipole moment** $\vec{\mu}_s$. (By intrinsic, we mean that $\vec{S}$ and $\vec{\mu}_s$ are basic characteristics of an electron, like its mass and electric charge.) Vectors $\vec{S}$ and $\vec{\mu}_s$ are related by:

$$\vec{\mu}_s = -\frac{e}{m}\vec{S}, \qquad (1)$$

in which *e* is the elementary charge (1.60 * 10^{-19} C) and *m* is the mass of an electron (9.11 * 10^{-31} kg). The minus sign means that $\vec{S}$ and $\vec{\mu}_s$ are oppositely directed. Spin $\vec{S}$ is different from the angular momenta in two respects:

1. Spin $\vec{S}$ itself cannot be measured. However, its component along any axis can be measured.

2. A measured component of $\vec{S}$ is *quantized,* which is a general term that means it is restricted to certain values. A measured component of $\vec{S}$ can have only two values, which differ only in sign.

Let us assume that the component of spin $\vec{S}$ is measured along the *z* axis of a coordinate system. Then the measured component S_z can have only the two values given by:

$$S_z = m_s \frac{h}{2\pi}, \qquad \text{for } m_s = \pm\tfrac{1}{2}, \qquad (2)$$

where m_s is called the *spin magnetic quantum number* and h (= 6.63 * 10^{-34} J.s) is the Planck constant, the ubiquitous constant of quantum physics. The signs given in equation (2) have to do with the direction of S_z along the z axis. When S_z is parallel to the z axis, m_s is $+\frac{1}{2}$ and the electron is said to be *spin up*. When S_z is antiparallel to the z axis, m_s is $-\frac{1}{2}$ and the electron is said to be *spin down*.

The spin magnetic dipole moment $\vec{\mu}_s$ of an electron also cannot be measured; only its component along any axis can be measured, and that component too is quantized, with two possible values of the same magnitude but different signs. We can relate the component $\mu_{s,z}$ measured on the z axis to S_z by rewriting equation (1)in component form for the z axis as:

$$\mu_{s,z} = -\frac{e}{m} S_z.$$

Substituting for S_z from equation (2) then gives us:

$$\mu_{s,z} = \pm \frac{eh}{4\pi m},$$

where the plus and minus signs correspond to $\mu_{s,z}$ being parallel and antiparallel to the z axis, respectively. The quantity on the right is the *Bohr magneton* μ_B:

$$\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24} \text{ J/T} \quad \text{(Bohr magneton).}$$

Spin magnetic dipole moments of electrons and other elementary particles can be expressed in terms of μ_B. For an electron, the magnitude of the measured z component of $\vec{\mu}_s$ is:

$$|\mu_{s,z}| = 1\mu_B.$$

Energy. When an electron is placed in an external magnetic field $\vec{B}_{ext}$, an energy U can be associated with the orientation of the electron's spin magnetic dipole moment $\vec{\mu}_s$ just as an energy can be associated with the orientation of the magnetic dipole moment $\vec{\mu}$ of a current loop placed in $\vec{B}_{ext}$. The orientation energy for the electron is:

$$U = -\vec{\mu}_s \cdot \vec{B}_{ext} = -\mu_{s,z} B_{ext},$$

where the z axis is taken to be in the direction of $\vec{B}_{ext}$.

If we imagine an electron to be a microscopic sphere (which it is not), we can represent the spin $\vec{S}$, the spin magnetic dipole moment $\vec{\mu}_s$, and the associated magnetic dipole field as in Figure (1).Although we use the word "spin" here, electrons do not spin like tops. How, then, can something have angular momentum without actually rotating? Again, we would need quantum physics to provide the answer.

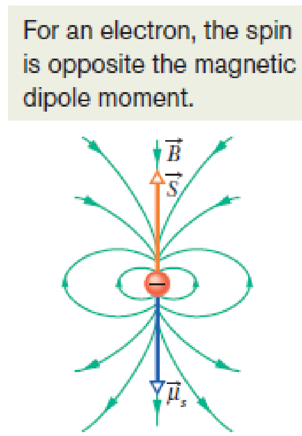

Figure (1): The spin $\vec{S}$, spin magnetic dipole moment $\vec{\mu}_s$, and magnetic dipole field $\vec{B}$ of an electron represented as a microscopic sphere.

Protons and neutrons also have an intrinsic angular momentum called spin and an associated intrinsic spin magnetic dipole moment. For a proton those two vectors have the same direction, and for a neutron they have opposite directions. We shall not examine the contributions of these dipole moments to the magnetic fields of atoms because they are about a thousand times smaller than that due to an electron.

3. Orbital Magnetic Dipole Moment

When it is in an atom, an electron has an additional angular momentum called its **orbital angular momentum** $\vec{L}_{orb}$. Associated with $\vec{L}_{orb}$ is an **orbital magnetic dipole moment** $\vec{\mu}_{orb}$; the two are related by:

$$\vec{\mu}_{\text{orb}} = -\frac{e}{2m}\vec{L}_{\text{orb}}. \quad (3)$$

The minus sign means that $\vec{\mu}_{orb}$ and $\vec{L}_{orb}$ have opposite directions.

Orbital angular momentum $\vec{L}_{orb}$ cannot be measured; only its component along any axis can be measured, and that component is quantized. The component along, say, a *z* axis can have only the values given by:

$$L_{\text{orb},z} = m_\ell \frac{h}{2\pi}, \qquad \text{for } m_\ell = 0, \pm 1, \pm 2, \ldots, \pm (\text{limit}), \tag{4}$$

in which m_ℓ is called the *orbital magnetic quantum number* and “limit” refers to some largest allowed integer value for m_ℓ. The signs in equation (4)have to do with the direction of $\vec{L}_{orb,z}$ along the *z* axis.

The orbital magnetic dipole moment $\vec{\mu}_{orb}$ of an electron also cannot itself be measured; only its component along an axis can be measured, and that component is quantized. By writing equation (3)for a component along the same *z* axis as above and then substituting for $L_{orb,z}$ from equation (4), we can write the *z* component $\mu_{orb,Z}$ of the orbital magnetic dipole moment as:

$$\mu_{\text{orb},z} = -m_\ell \frac{eh}{4\pi m}$$

and, in terms of the Bohr magneton, as:

$$\mu_{\text{orb},z} = -m_\ell \mu_{\text{B}}.$$

When an atom is placed in an external magnetic field $\vec{B}_{ext}$, an energy U can be associated with the orientation of the orbital magnetic dipole moment of each electron in the atom. Its value is:

$$U = -\vec{\mu}_{\text{orb}} \cdot \vec{B}_{\text{ext}} = -\mu_{\text{orb},z} B_{\text{ext}},$$

where the *z* axis is taken in the direction of $\vec{B}_{ext}$.

Although we have used the words “orbit” and “orbital” here, electrons do not orbit the nucleus of an atom like planets orbiting the Sun. How can an electron have an orbital angular momentum without orbiting in the common meaning of the term? Once again, this can be explained only with quantum physics.

4. Loop Model for Electron Orbits

We can obtain equation (3) with the nonquantum derivation that follows, in which we assume that an electron moves along a circular path with a radius that is much larger than an atomic radius (hence the name “loop model”). However, the derivation does not apply to an electron within an atom (for which we need quantum physics).

We imagine an electron moving at constant speed *v* in a circular path of radius *r*, counterclockwise as shown in Figure (2). The motion of the negative charge of the electron is equivalent to a conventional current *i* (of positive charge) that is clockwise, as also shown in Figure (2). The magnitude of the orbital magnetic dipole moment of such a *current loop* is obtained from equation (6) with $N = 1$:

$$\mu_{\text{orb}} = iA, \qquad (5)$$

where *A* is the area enclosed by the loop. The direction of this magnetic dipole moment is, from the right-hand rule, downward in Figure (2).

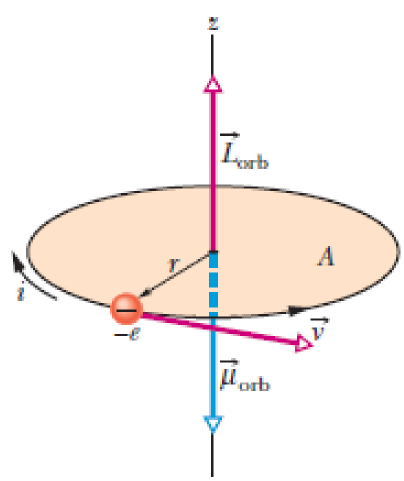

Figure (2): An electron moving at constant speed *v* in a circular path of radius *r* that encloses an area *A*. The electron has an orbital angular momentum $\vec{L}_{orb}$ and an associated orbital magnetic dipole moment $\vec{\mu}_{orb}$. A clockwise current *i* (of positive charge) is equivalent to the counterclockwise circulation of the negatively charged electron.

To evaluate equation (5), we need the current i. Current is, generally, the rate at which charge passes some point in a circuit. Here, the charge of magnitude e takes a time $T = 2\pi r/v$ to circle from any point back through that point, so:

$$i = \frac{\text{charge}}{\text{time}} = \frac{e}{2\pi r/v}.$$

Substituting this and the area $A = \pi r2$ of the loop into equation (5) gives us

$$\mu_{\text{orb}} = \frac{e}{2\pi r/v}\pi r^2 = \frac{evr}{2}. \qquad (6)$$

To find the electron's orbital angular momentum $\vec{L}_{orb}$, we use equation, $\vec{\ell} = m\,(\vec{r} \times \vec{v})$.Because $\vec{r}$ and $\vec{v}$ are perpendicular, $\vec{L}_{orb}$ has the magnitude:

$$L_{\text{orb}} = mrv \sin 90° = mrv. \qquad (7)$$

The vector $\vec{L}_{orb}$ is directed upward in Figure (2). Combining equation (6 and 7), generalizing to a vector formulation, and indicating the opposite directions of the vectors with a minus sign yield:

$$\vec{\mu}_{\text{orb}} = -\frac{e}{2m}\vec{L}_{\text{orb}},$$

which is equation (3). Thus, by "classical" (nonquantum) analysis we have obtained the same result, in both magnitude and direction, given by quantum physics. You might wonder, seeing as this derivation gives the correct result for an electron within an atom, why the derivation is invalid for that situation. The answer is that this line of reasoning yields other results that are contradicted by experiments.