



محاضرات إحصاء طبي / نظري 3

المرحلة الثانية

قسم التخدير

م. د. ايناس رضا علي

Properties of Mean

1. It requires at least the interval scale.
2. All values are used.
3. It is unique.
4. It is easy to calculate and allow easy mathematical treatment.
5. The sum of the deviations from the mean is 0.
6. The arithmetic mean is the only measure of central tendency where the sum of the deviations of each value from the mean is zero.
7. It is easily affected by extremes, such as very big or small numbers in the set (non-robust).

How Extremes Affect the Mean?

1. The mean of the values 1,1,1,1,100 is 20.8.
2. However, 20.8 does not represent the typical behavior of this data set!
3. Extreme numbers relative to the rest of the data are called outliers!
4. Examination of data for possible outliers serves many useful purposes, including
 - Identifying strong skew in the distribution.
 - Identifying data collection or entry errors.
 - Providing insight into interesting properties of the data.

Mean for Grouped Data:

The formula provided above is being used when the number of values is small. If the number of values is large, they are grouped into a frequency distribution. In the case of grouped data when the data is arranged in the form of frequency distribution, all the values falling in a class are assumed to be equal to the class mark or midpoint. If the X_1, X_2, \dots, X_k are the class marks with f_1, f_2, \dots, f_k as the corresponding class frequencies, the sum of the values in the first class would be f_1X_1 , in the second class f_2X_2 , and so on the sum of the values in kth class would be f_kX_k . Hence, the sum of the values in all the k classes would be

$$f_1X_1 + f_2X_2 + \dots + f_kX_k = \sum fX$$

The total number of values is the sum of the class frequencies, as follows.

$$f_1 + f_2 + \dots + f_k = \sum f$$

Since, the mean is the value obtained by dividing the sum of the values by their number, the mean for grouped data is calculated by the following formula.

$$mean = \bar{X} = \frac{\sum fX}{\sum f}$$

Example:

Find the mean weight of 120 weight students at a university from the following frequency distribution below Table.

Weight (lb)	Class Mark (X)	Frequency (f)	fX
110 – 119	114.5	1	114.5
120 – 129	124.5	4	498
130 – 139	134.5	17	2286.5
140 – 149	144.5	28	4046
150 – 159	154.5	25	3862.5
160 – 169	164.5	18	2961
170 – 179	174.5	13	2268.5
180 – 189	184.5	6	1107
190 – 199	194.5	5	972.5
200 – 209	204.5	2	409
210 -219	214.5	1	214.5
		$\sum f = 120$	$\sum fX = 18740$

$$mean = \bar{X} = \frac{\sum fX}{\sum f} = \frac{18740}{120} = 156.7 \text{ pounds}$$

Weighted Mean

What is the Weighted Mean?

The weighted mean is a type of mean that is calculated by multiplying the weight (or probability) associated with a particular event or outcome with its associated quantitative outcome and then summing all the products together. It is very useful when calculating a theoretically expected outcome where each outcome has a different probability of occurring, which is the key feature that distinguishes the weighted mean from the arithmetic mean.

Weighted Average Formula

$$\text{Weighted Average} = \frac{\text{Sum of weighted terms}}{\text{total number of terms}}$$

$$\bar{x} = \frac{w_1x_1 + w_2x_2 + w_3x_3 + \cdots + w_ix_i + \cdots + w_nx_n}{w_1 + w_2 + w_3 + \cdots + w_i + \cdots + w_n} = \frac{\sum_{i=1}^n w_ix_i}{\sum_{i=1}^n w_i}$$

If data is from a population, μ replaces \bar{x} .

$$\bar{x} = \frac{\sum w_ix_i}{\sum w_i}$$

Numerator:
sum of the weighted
data values

Denominator:
sum of the
weights

where:

x_i = value of observation i

w_i = weight for observation i

NOTE: When we use the simple mean or simple average, we are giving equal weight (1/n) to each observation or data point and the sum of the weights is equal to 1.

The weighted mean is used in:

- Construction of index numbers.
- Comparison of results of two or more universities where the number of students differs.
- Computation of standardized death and birth rates.

Example

Find the mean of the following data set.

1	1	1	1		
10	10	10	10	10	
5	5	5	5	5	5

Solution.

Use the Weighted Mean formula.

The w terms are the weights.

$$\text{Weighted Average} = \frac{\text{Sum of weighted terms}}{\text{total number of terms}}$$

$$= \frac{w_1 \cdot x_1 + w_2 \cdot x_2 + \dots + w_n \cdot x_n}{w_1 + w_2 + \dots + w_n}$$

$$\begin{aligned} \text{Weighted Mean} &= \frac{4 \cdot 1 + 5 \cdot 10 + 6 \cdot 5}{15} \\ &= \frac{4 + 50 + 30}{15} \\ &= \frac{84}{15} \\ &= 5.6 \end{aligned}$$

The numbers in red are the weights.

Geometric mean

In mathematics, the geometric mean is a mean or average, which indicates the central tendency or typical value of a set of numbers by using the product of their values (as opposed to the arithmetic mean which uses their sum). The geometric mean is defined as the nth root of the product of n numbers, i.e., for a set of numbers x_1, x_2, \dots, x_n , the geometric mean is defined as

$$\text{Geometric mean} = \sqrt[n]{x_1 \cdot x_2 \cdot \dots \cdot x_n}$$

GEOMETRIC MEAN

roots and multiplication

multiply numbers together and then find the n^{th} root of the numbers such that the n^{th} root is equal to the amount of numbers you multiplied

$$\begin{array}{c} \sqrt[3]{x_1 \cdot x_2 \cdot x_3} \\ \sqrt[5]{x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5} \\ \sqrt[11]{x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5 \cdot x_6 \cdot x_7 \cdot x_8 \cdot x_9 \cdot x_{10} \cdot x_{11}} \\ \sqrt[4]{x_1 \cdot x_2 \cdot x_3 \cdot x_4} \end{array}$$

For example, if you multiply three numbers, the geometric mean is the third root of the product of those three numbers. The geometric mean of five numbers is the fifth root of their product. Suppose we said we found the geometric mean using the 11th root of the numbers. That tells you that 11 numbers were multiplied together. To find the geometric mean of four numbers, what root would we take? The fourth root, of course.

How to Find the Geometric Mean

We will start with an easy example using only two numbers, 4 and 9. What is the geometric mean of 4 and 9?

Multiply $4 \cdot 9$

Then find the square root of their product (because you only multiplied two numbers):

How to find the **GEOMETRIC MEAN**

What is the geometric mean of 4 and 9?

$$\sqrt[2]{4 \cdot 9} = \sqrt[2]{36} = \underline{\underline{6}}$$

Uses for the Geometric Mean

Anytime we are trying to calculate average rates of growth where growth is determined by multiplication, not addition, we need the geometric mean. This connects geometric mean to economics, financial transactions between banks and countries, interest rates, and personal finances.

Harmonic mean

The harmonic mean can be expressed as the reciprocal of the arithmetic mean of the reciprocals of the given set of observations.

$$\text{Harmonic mean} = \frac{n}{\sum \frac{1}{x_i}}$$

Where:

n : the number of the values in a dataset

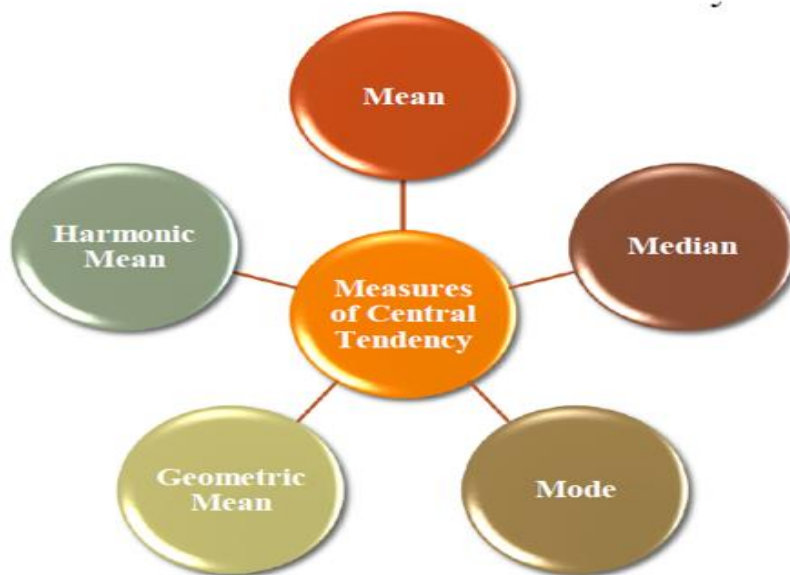
x_i : the point in a dataset

Example

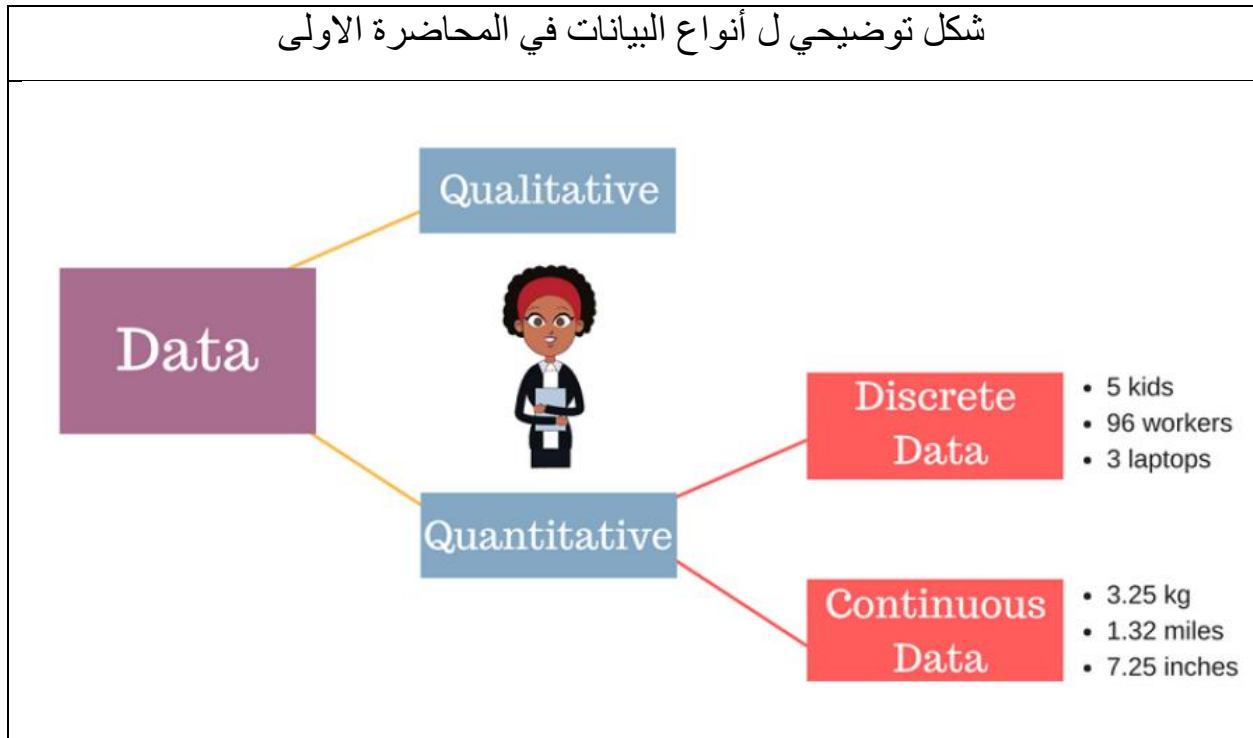
consider 2, 3, 5, 7, and 60 with a number of observations as 5 find Harmonic mean.

$$\begin{aligned}\text{mean} &= \frac{2+3+5+7+60}{5} \\ &= \frac{77}{5} \\ &= 15.4\end{aligned}$$

$$\begin{aligned}\text{Harmonic Mean} &= \frac{n}{\left(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}\right)} \\ &= \frac{5}{\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{60}\right)} \\ &= \frac{5}{(0.5 + 0.33 + 0.2 + 0.14 + 0.017)} \\ &= \frac{5}{1.187} \\ &= 4.21\end{aligned}$$



شكل توضيحي ل أنواع البيانات في المحاضرة الاولى



Examples of discrete data:

1. The number of students in a class.
2. The number of workers in a company.
3. The number of parts damaged during transportation.
4. Shoe sizes.
5. Number of languages an individual speaks.
6. The number of home runs in a baseball game.
7. The number of test questions you answered correctly.
8. Instruments in a shelf.
9. The number of siblings a randomly selected individual has.

Examples of continuous data:

1. The amount of time required to complete a project.
2. The height of children.
3. The amount of time it takes to sell shoes.
4. The amount of rain, in inches, that falls in a storm.
5. The square footage of a two-bedroom house.
6. The weight of a truck.
7. The speed of cars.
8. Time to wake up.