A probability model

A probability model also requires a collection of events,

which are subets of S to which probabilities can be assigned.

For the above weather example, the subsets {rain}, {snow}, {rain, snow}, {rain, clear}, {rain, snow, clear}, and even the empty set Ø = { }, are all examples of subsets of S that could be events.

are all examples of subsets of S that could be events. Note that here the comma means “or”; thus, {rain, snow} is the event that it will rain or snow.

I will generally assume that all subsets of S are events.

NOTE:

In fact, in complicated situations there are some technical restrictions on what subsets can or cannot be events, according to the mathematical subject of measure theory.

But we will not concern ourselves with such technicalities here.)

Finally, and most importantly, a probability model requires a probability measure, usually written P. This probability measure must assign, to each event A, a probability P(A).

We require the following properties:

1. P(A) is always a nonnegative real number, between 0 and 1 inclusive.

2. $P(\emptyset) = 0$, i.e., if A is the empty set $\emptyset$, then $P(A) = 0$.

3. $P(S) = 1$, i.e., if A is the entire sample space S, then $P(A) = 1$.

4. P is (countably) additive, meaning that if $A1, A2, \ldots$ is a finite or countable sequence of disjoint events,

then $P(A1 \cup A2 \cup \cdots) = P(A1) + P(A2) + \cdots$.

A- The first of these properties says that we shall measure all probabilities on a scale from 0 to 1, where 0 means impossible and 1 (or 100%) means certain.

B- The second property says the probability that nothing happens is 0; in other words, it is impossible that no outcome will occur.

C- The third property says the probability that something happens is 1; in other words, it is certain that some outcome must occur.

The fourth property is the most subtle.

It says that we can calculate probabilities of complicated events by adding up the probabilities of smaller events, provided those smaller events are disjoint and together contain the entire complicated event.

Note that events are disjoint if they contain no outcomes in common

Example:

{rain} and {snow, clear} are disjoint, whereas {rain} and {rain, clear} are not disjoint.

(Im are assuming for simplicity that it cannot both rain and snow tomorrow.)

P({rain}) + P({snow, clear}) = P({rain, snow, clear}), but do not expect to have P({rain}) + P({rain, clear}) = P({rain, rain, clear}) (the latter being the same as P({rain, clear})).

now formalize the definition of a probability model.
Definition 1.2.1 A probability model consists of a nonempty set called the sample space S;

a collection of events that are subsets of S; and a probability measure P assigning a probability between 0 and 1 to each event, with $P(\emptyset) = 0$ and $P(S) = 1$ and with

Probability Models

EXAMPLE 1.

Consider again the weather example, with S = {rain, snow, clear}. Suppose that the probability of rain is 40%, the probability of snow is 15%, and the probability of a clear day is 45%. We can express this as P({rain}) = 0.40, P({snow}) = 0.15, and P({clear}) = 0.45. For this example, of course P(∅) = 0, i.e., it is impossible that nothing will happen tomorrow.

Also P({rain, snow, clear}) = 1,

because we are assuming that exactly one of rain, snow, or clear must occur tomorrow.

(To be more realistic, we might say that we are predicting the weather at exactly 11:00 A.M. tomorrow.) Now, what is the probability that it will rain or snow tomorrow?

SOL :

, by the additivity property, we see that P({rain,snow}) = P({rain}) + P({snow}) = 0.40 + 0.15 = 0.55

thus conclude that, as expected, there is a 55% chance of rain or snow tomorrow.

EXAMPLE

Suppose your candidate has a 60% chance of winning an election in progress.

Then S = {win, lose}, with P(win) = 0.6 and P(lose) = 0.4.

Note that P(win)+P(lose) = 1.

EXAMPLE.3

Suppose we flip a fair coin, which can come up either heads (H) or tails (T) with equal probability.

SOL:

Then S = {H, T }, with P(H) = P(T) = 0.5. Of course, P(H)+P(T) = 1.

EXAMPLE :

Suppose we flip three fair coins in a row and keep track of the sequence of heads and tails that result.

SOL :

Then S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT }. Furthermore, each of these eight outcomes is equally likely.

Thus, P(HHH) = 1/8, P(TTT) = 1/8, etc. Also, the probability that the first coin is heads and the second coin is tails, but the third coin can be anything, is equal to the sum of the probabilities of the events HTH and HTT , i.e., P(HTH) + P(HTT) = 1/8 + 1/8 = 1/4.

EXAMPLE :

Suppose we flip three fair coins in a row but care only about the number of heads that result. Then S = {0, 1, 2, 3}.

SOL:

the probabilities of these four outcomes are not all equally likely; we will see later that in fact

P(0) = P(3) = 1/8,

P(1) = P(2) = 3/8