Analytic Mechanics

Fourth lecture

Dynamics of a particle, Rectilinear Motion

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1. *Newton Law of motion*

Three fundamental laws of motion:

I. Law 1: An object will remain in its state of motion, be it at rest or moving with constant velocity, unless a net external force is exerted on the object.

II. Law 2: An object's acceleration is proportional to the net force exerted on the object, inversely proportional to the mass of the object, and in the same direction as the net force exerted on the object.

III. Law 3: If one object exerts a force on another object, the second object exerts a force on the first object that is equal in magnitude and opposite in direction.

2. *Newton's First Law: Inertial Reference Systems*

**The first law describes property shared by all matter, namely, (inertia) inertia is the resistance of all matter to having its motion changed.

** The law states that a moving body travels in a straight line with constant speed unless some influence called force prevents the body from doing so.

** whether or not a body move in a straight line with constant speed depends not only on external influence (force) but also on the particular reference system that is used to describe the motion.

** The first law definition of a particular kind of reference system called Newtonian or inertial reference system.

***Example* [*1*]:**

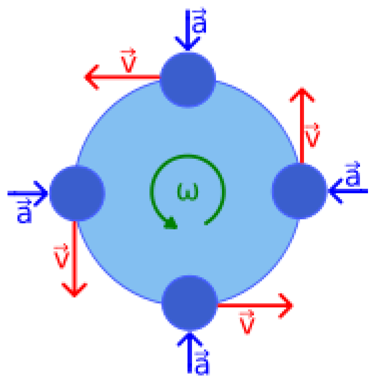
Calculate the centripetal acceleration relative to the acceleration due to gravity g, of

(a) a point on the surface of the Earth's equator (the radius of the Earth is $R_E = 6.4 \times 10^3$ km)
(b) the Earth in its orbit about the Sun (the radius of the Earth's orbit is $a_E = 150 \times 10^6$ km)
(c) the Sun in its rotation about the center of the galaxy (the radius of the Sun's orbit about the center of the galaxy is $R_G = 2.8 \times 10^4$ LY. Its orbital speed is $v_G = 220$ km/s)

**Centripetal acceleration: the acceleration of a body in uniform circular motion, its direction is toward the center. $a_c = \frac{v^2}{R}$, $v = \omega R$

**The centripetal acceleration of a point rotating in a circle of radius R is given by

$$a_c = \omega^2 R = \left(\frac{2\pi}{T}\right)^2 R = \frac{4\pi^2 R}{T^2}$$



**Where T is period of one complete rotation.

Thus, relative to g we have

$$\frac{a_c}{g} = \frac{4\pi^2 R}{gT^2}$$

(a) $\frac{a_c}{g} = \frac{4\pi^2(6.4\times10^6\,\mathrm{m})}{9.8\,\mathrm{m\cdot s^{-2}}(3.16\times10^7\,\mathrm{s})^2} = 3.4\times10^{-3}$
(b) 6×10^{-4}
(c) 1.5×10^{-12}

2. Mass and Force: Newton's Second and Third Laws

**The quantitative measure of inertia is called mass. Newton's second law:

$$\bar{F} = m\frac{\overline{dv}}{dt}$$

**Newton's third law, namely, that two interacting bodies exert equal and opposite forces upon one another.

$$\bar{F}_a = -\bar{F}_b$$

Linear Momentum: The product of mass and velocity

P = mv

$$\therefore \bar{F} = \frac{\overline{dp}}{dt}$$

$$\therefore 3rd\ law => \frac{\overline{dp_A}}{dt} = -\frac{\overline{dp_B}}{dt}$$

$$\frac{\overline{dp_A}}{dt} = -\frac{\overline{dp_B}}{dt} => \frac{d}{dt}(\overline{P_A} + P_B) = 0$$

$$\therefore\ \overline{P_A} + \overline{P_B} => canstant$$

**The total linear momentum of two interacting bodies always remains constant.

Example [2]:

A spaceship of mass M is traveling in deep space with velocity v_i= 20 km/s relative to the Sun. It ejects a rear stage of mass 0.2 M with a relative speed u = 5km/s what then is the velocity of the spaceship?

The total linear momentum is conserved

$$\mathbf{P}_f = \mathbf{P}_i$$

Where the subscripts i and f refer to initial and final values respectively.

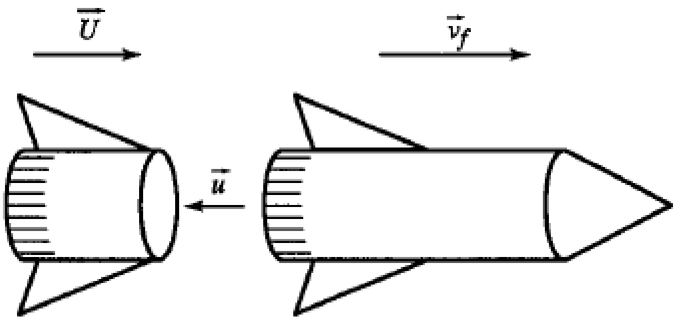
$$P_i = Mv_i$$

The total momentum of the system after ejection is then

$$P_f = 0.20\, MU + 0.80\, Mv_f$$

$$u = v_f - U$$

$$U = v_f - u$$



Use the conservation of momentum condition, we find

$$0.20\, M(v_f - u) + 0.8\, Mv_f = Mv_i$$

$$v_f = v_i + 0.2u = 20 \text{ km/s} + 0.20\,(5 \text{ km/s}) = 21 \text{ km/s}$$

3. Motion of a particle

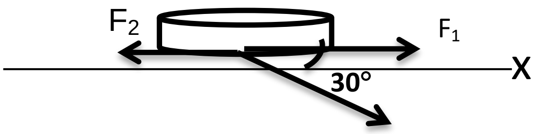
$$\vec{F} = \sum \vec{F}i = m\frac{d^2r}{dt^2} = m\vec{a}$$

$$\overrightarrow{F_x} = \sum F_x = M\ddot{x}, \overrightarrow{F_y} = ZF_{iy} = M\ddot{y}$$

$$\overrightarrow{F_z} = ZF_{iz} = M\ddot{z}$$

***Example* [3]:** The puck mass is m=0.2 kg, the force act on a puck that moves over frictionless ice along an x-axis. Forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are directed along the x-axis and have magnitudes F_1 = 4N and F_2 = 2N. Force $\overrightarrow{F_3}$ is directed at angle θ = 30° has magnitude F_3 = 1N, what is the acceleration on the puck?

$\overrightarrow{F_{net}} = m\vec{a}$



The motion along x-axis

$$\overrightarrow{F_{net,x}} = m\vec{a}_x$$

$$\vec{a}_x = \frac{\overrightarrow{F_{net,x}}}{m} = \frac{F_1 + F_{3,X} - F_2}{m} = \frac{F_1 + F_3 cos\theta - F_2}{m}$$

$$= \frac{4 + 1cos30 - 2}{0.2}$$

= **14.33 m/s^2**

4. Rectilinear Motion

$$F(x, x, t) = M\ddot{x}$$

$$F = m\frac{dv}{dt}\ [\,F = ma] => \ a = \frac{F}{m}$$

$$\frac{dv}{dt} = \frac{f}{m} = a$$

$$\therefore \int_{vo}^{t} dv = \int_{0}^{t} adt$$

$$v = v_0 + at$$

$$v = at + v_0$$

$$\frac{dx}{dt} = at + v_0$$

$$\therefore \int_{x_0}^{x} dx = \int_{0}^{x} atdt + \int_{0}^{1} v_0 dt$$

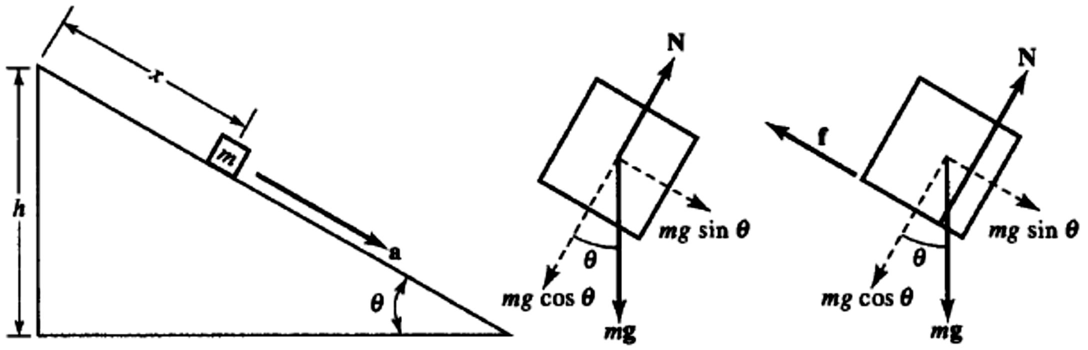
$$= x = X_0 + \frac{1}{2}at^2 + v_0 t$$

$$2a(x - x_0) = v^2 - v_0^2$$

**The equation above that the equation of uniformly accelerated motion.

***Example* [4]:**

Consider a block that is free to slide down a smooth, frictionless plane that is inclined at an angle θ to the horizontal, as shown in Figure (1). If the height of the plane is h and the block is released from rest at the top, what will be its speed when it reaches the bottom?



We choose a coordinate system whose positive x-axis points down the plane as shown in the figure. The only force along the x direction is the component of gravitational force, mg sinθ, use the equations of motion,

$$\ddot{x} = a = \frac{F_x}{m} = g \sin\theta$$

$$x - x_0 = \frac{h}{\sin\theta}$$

$$v^2 = 2(g \sin\theta)\left(\frac{h}{\sin\theta}\right) = 2gh$$

Suppose that, instead of being smooth, the plane is rough; that is, it exerts a frictional force f on the particle. Then the net force in the x direction is equal to mg sineθ-f Now, for sliding contact it is found that the magnitude of the frictional force is proportional to the magnitude of the normal force N; that is,

$$f = \mu_K N$$

Where the constant of proportionality μ_k is known as the coefficient of sliding or kinetic friction. In the example under discussion, the normal force, as shown in the figure, is equal to mg cos θ; hence,

$$f = \mu_K mg \cos\theta$$

Consequently, the net force in the x direction is equal to

$$mg \sin\theta - \mu_K mg \cos\theta$$

$$\ddot{x} = \frac{F_x}{m} = g(\sin\theta - \mu_k \cos\theta)$$

The speed of the particle increases if the expression in parentheses is positive