Differential Equations

Differential Equations: is a mathematical equations that relates some functions with its derivatives.

Equation Order: Differential Equations are described by their order, determined by term with highest derivatives.

Example: An equation containing only first derivatives is (First order differential equation), an equation containing only second derivatives is (second order differential equation) and so on.

Ex: State D.E. order

1) $\frac{du}{dx} = cu + x^2$ (First order differential equation)

2) $\frac{d^2u}{dx^2} + w^2u = 0$ (Second order differential equation)

3) $\frac{du}{dt} = 6u\frac{du}{dx} - \frac{d^3u}{dx^3}$ (Third order differential equation)

Differential Equations Degree: Is the power of its highest derivative.

Ex: State D.E. order and degree?

1) $\frac{dy}{dx} + x - 7y = 0$ (First order, first degree)

2) $\frac{d^2y}{dx^2} = 5x - 3xy + 7$ (Second order, first degree)

3) $\frac{d^3y}{dx^3} - 2\left(\frac{dy}{dx}\right)^5 + 3y = 0$ (Third order, first degree)

4) $\frac{du}{dt} = 6u\frac{du}{dx} - \left(\frac{d^3u}{dx^3}\right)^2$ (Third order, Second degree)

1) Separation of Variables

1st Order DE - Separable Equations

The differential equation $M(x,y)dx + N(x,y)dy = 0$ is separable if the equation can be written in the form:

$$\boxed{f_1(x)g_1(y)dx + f_2(x)g_2(y)dy = 0}$$

Solution :

1. Multiply the equation by integrating factor: $\boxed{\dfrac{1}{f_2(x)g_1(y)}}$
2. The variable are separated :

$$\boxed{\frac{f_1(x)}{f_2(x)}dx + \frac{g_2(y)}{g_1(y)}dy = 0}$$

3. Integrating to find the solution:

$$\boxed{\int \frac{f_1(x)}{f_2(x)}dx + \int \frac{g_2(y)}{g_1(y)}dy = C}$$

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Differential Equations
(D.E.)

- a differential equation is a mathematical equation that relates some function with it's derivatives.

- Equation order:- differential equations are described by their order, determined by the term with highest derivatives. An equation containing only first derivatives is (first order differential equation), an equation containing second derivative is (second - order differential eq), and so on.

Ex:

1- $\frac{du}{dX} = cu + X^2$ (first order differential eq).

2- $\frac{d^2u}{dX^2} + w^2 u = 0$ (second order differential eq).

3- $\frac{du}{dt} = 6u \cdot \frac{du}{dX} - \frac{d^3u}{dX^3}$ (third order differential eq)

- Equation dgree: is the power of its highest derivative.

Ex:

1- $\frac{dy}{dx} + X - 7y = 0$ first order, first dgree.

2- $\frac{d^2y}{dX^2} = 5X - 3Xy + 7$ Second order, first dgree.

3- $\left(\frac{d^3y}{dx^3}\right)^2 - 2\left(\frac{dy}{dx}\right)^5 + 3y = 0$ third order, second dgree

4- $x^2\left(\frac{dy}{dx}\right)^4 + \left(\frac{d^3y}{dx^3}\right)^2 + 2\frac{d^2y}{dx^2} = 0$ third order, second dgree

5- $(\bar{\bar{\bar{y}}})^3 - 2\bar{y} + 8y = x^3 + \cos x$ third order, third dgree.

H.W./ find the order and dgree of following D.E.

① $\frac{d^2y}{dx^2} + x.\frac{dy}{dx} - 5y = 0$

② $(\bar{\bar{\bar{y}}})^3 - 2\bar{y} + 8y = x^3 + \cos x$

③ $\left(\frac{d^3y}{dx^3}\right)^2 - 2\left(\frac{dy}{dx}\right)^5 + 3y = 0$

Solve The D-E.:

Ex1: Prove the $(y = x^2 + 3x)$ is an answer for D.E. $x\bar{y} = x^2 + y$?

Sol:- $y = x^2 + 3x$

$\bar{y} = 2x + 3$

نأخذ طرفي المعادلة التفاضلية :

الطرف الأيسر $= LHS = x\bar{y} = x(2x+3)$

$= 2x^2 + 3x$

الطرف الأيمن $= RHS = x^2 + y = x^2 + (x^2 + 3x).$

$= 2x^2 + 3x$ $\therefore LHS = RHS$

EX_2: Prove that $y = x\ln|x| - x$ is an answer for the D.E. $\left(x\frac{dy}{dx} = x + y\right)$?

Sol: $y = x\ln|x| - x$

$\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln|x| - 1$

$\frac{dy}{dx} = 1 + \ln|x| - 1 = \ln|x| \longrightarrow \boxed{\frac{dy}{dx} = \ln x}$

نأخذ المعادلة التفاضلية:

$x\frac{dy}{dx} = x + y$

الطرف الأيسر $= LHS = x \cdot \frac{dy}{dx} = x \cdot \ln|x|$

الطرف الأيمن $= RHS = x + y = \cancel{x} + x\ln|x| - \cancel{x}$

$= x \cdot \ln|x|$

$\therefore LHS = RHS$

So, $y = x\ln|x| - x$ answer for D.E.

EX_3: $y = 3\cos 2x + 2\sin 2x$ is an answer for D.E. $y'' + 4y = 0$?

Sol: $y' = -6\sin 2x + 4\cos 2x$

$y'' = -12\cos 2x - 8\sin 2x$

$\ddot{y} + 4y = 0$

$(-12\cos 2x - 8\sin 2x) + 4(3\cos 2x + 2\sin 2x)$.

$= -12\cos 2x - 8\sin 2x + 12\cos 2x + 8\sin 2x$

$= 0$

So, $y = 3\cos 2x + 2\sin 2x$ is an answer for D.E.

H.W. :-

① $y^2 = 3x^2 + x^3$ is answer for $y\ddot{y} + (\dot{y})^2 - 3x = 5$?

② $y = e^{2x} + e^{-3x}$ is answer for $\ddot{y} + \dot{y} - 6y = 0$?

③ $y = \cot x$ is answer for $\frac{d^2y}{dx^2} = 2\csc^2 x \cdot \cot x$?

Solution of D.E.

1- Solve D.E. by seperation of variables:

general form: $\boxed{f(x)\,dx = g(y)\,dy}$

Ex1: Solve $(dx + xy\,dy = y^2\,dx + y\,dy)$?

Sol:- $dx - y^2\,dx + xy\,dy - y\,dy = 0$

$(1-y^2)\,dx + (x-1)y\,dy = 0 \,] \div (1-y^2)(x-1)$

$\frac{dx}{x-1} + \frac{y\,dy}{1-y^2} = 0$

$\frac{1}{2}\ln(1-y^2) = \ln(x-1) + c$.

Ex2: $4xy^2\,dx + (x^2+1)\,dy = 0$?

Sol:- $4xy^2\,dx + (x^2+1)\,dy = 0 \,]\, \div y^2(x^2+1)$

$$\frac{4x}{x^2+1}\,dx + \frac{dy}{y^2} = 0 \qquad (\text{بالتكامل})$$

$$2\ln(x^2+1) - \frac{1}{y} = c$$

Ex3: Solve $\bar{y} - x\sqrt{y} = 0$

Sol:- $\frac{dy}{dx} - x\,y^{1/2} = 0$

$dy = x\,y^{1/2}\,dx \,]\, \div y^{1/2}$

$$\frac{dy}{y^{1/2}} = x\,dx \qquad (\text{بالتكامل})$$

$$\int y^{-\frac{1}{2}}\,dy = \int x\,dy \rightarrow 2\sqrt{y} = \frac{x^2}{2} + c$$