Lecture 3
Third stage

Mathematical Representation of W.F

By

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التمثيل الرياضي لدالة الموجة Mathematical Representation of W.F

بما ان المجال المادي المصاحب لحركة الجسيم يمكن التعبير عنه بموجات واقفة حيث يمكن تمثيل دالة الموجة بالصيغة الرياضية التالية:

$$\Psi_{(x,t)}=\Psi_0 e^{i(\frac{2\pi}{\lambda}x-\omega t)}$$

- $p=\hbar k$, $E=\hbar\omega$, $K=2\pi/\lambda$
- $\Psi_{(X,t)}=\Psi_0 e^{\frac{i}{\hbar}(PxX-Et)}$ .....(1)

In Three Dimension

$$\Psi_{(r,t)}=\Psi_0 e^{\frac{i}{\hbar}(P.r-Er)}$$

معادلة Time Dependent Schrodinger Equation (T.D.S.E)
شرود نكر المعتمدة على الزمن

باشتقاق معادلة معادلة (١) بالنسبة x

$$\frac{\partial \Psi(x,t)}{\partial x} = \frac{\partial}{\partial x}\left(\Psi_0 e^{\frac{i}{\hbar}(PxX-Et)}\right) = \Psi_0\, e^{\frac{i}{\hbar}(PxX-Et)}(\frac{i}{\hbar}Px)$$

$$\frac{\partial \Psi(x,t)}{\partial x} = i/\hbar\, Px\, \Psi(x,t)$$

نضرب طرفي المعادلة في -iħ

$$-i\hbar \frac{\partial \Psi(x,t)}{\partial x} = i/\hbar\, Px\, \Psi(x,t) \qquad \dots\dots(2)$$

نشتق معادلة (2) بالنسبة ل (x)

$$-\hbar\text{^}2\ \frac{\partial\text{^}2\Psi(x,t)}{\partial\text{^}2x} = P\text{^}2x\,\Psi(x,t) \qquad \dots\dots(3)$$

نشتق معادلة (1) بالنسبة للزمن (t)

$$\frac{\partial \Psi(x,t)}{\partial t} = \frac{\partial}{\partial t}\left(\Psi_0 e^{\frac{i}{\hbar}(PxX-Et)}\right) = \Psi_0\, e^{\frac{i}{\hbar}(PxX-Et)}(-\frac{i}{\hbar}E)$$

نضرب طرفي المعادلة فيiħ

$$\frac{\partial \Psi(x,t)}{\partial x} = -i/\hbar\, E\, \Psi(x,t)$$

$$i\hbar\,\frac{\partial \Psi(x,t)}{\partial x} = E\,\Psi(x,t) \qquad \ldots\ldots(4)$$

❖*E= T+V*

$$E=\frac{P2}{2m}+ V_{(X)} \qquad \ldots*$$

نضرب طرفي المعادلة * بـــ $\Psi(x,t)$

$$E\,\Psi(x,t) = \frac{P\text{^}2\Psi(x,t)}{2m} + V(X)\Psi(x,t) \qquad \ldots.\,..(5)$$

بالتعويض عن (4) ، (3) في المعادلة (5) نحصل على العلاقة التالية:

$$i\hbar = \frac{\partial\Psi(x,t)}{\partial t} = -\frac{-\hbar\text{^}2}{2m}\ \frac{\partial\Psi(x,t)}{\partial x}$$

$$i\hbar\,\frac{\partial\Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\,\frac{\partial^{2\Psi(x,t)}}{\partial x^2} + v(x)\Psi(x,t) \qquad \ldots\ldots(6)$$

تدعى المعادلة (٦) بمعادلة شرودنكر المعتمدة على الزمن وهي معادلة ذات اهمية كبيرة في ميكانيك الكم وفي ثلاث ابعاد تصبح العلاقة (٦) بالصيغة الرياضية التالية :

In Three Dimension

$$i\hbar \frac{\partial \Psi(r,t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^{2\Psi(r,t)} + v(r)\Psi(r,t)$$

معادلة Time Independent Schrodinger Equation(T.I.S.E) شرودنكر الغير المعتمدة على الزمن

لايجاد معادلة شرودنكر الغير معتمدة على الزمن يمكن كتابة معادلة الموجة الواقفة على الشكل التالي:

$$\Psi(X,T) = \Psi_0 e^{\frac{i}{\hbar}(pxX - Et)}$$

$$=\Psi(t)\Psi(x)$$

$$\Psi(x,t) = e^{-\frac{i}{\hbar}Et}\ \Psi(x) \quad \dots\dots(7)$$

بتعويض المعادلة (٧) في المعادلة (٦) نحصل على :

$$i\hbar\ .\ (\frac{-i}{\hbar})\ E\Psi(x)\ e^{-\frac{i}{\hbar}Et} = -\frac{\hbar^2}{2m}$$

$$\frac{\partial\text{^}2\Psi(x,t)}{\partial x\text{^}2}\ e^{-\frac{i}{\hbar}Et} + v(x)\Psi(x)\ e^{-\frac{i}{\hbar}Et}$$

- $$-\frac{\hbar^2}{2m}\frac{\partial\text{^}2}{\partial x\text{^}2} + v(x)\Psi(x) = E\Psi(x) \qquad \dots\dots(8)$$

In Three Dimension

$$\nabla^{2\Psi(r)} + \frac{2m}{\hbar^2}[E - V(r)].\Psi(r) = 0$$

Normalized function

P(x,t)=Ψ* (x,t) Ψ(x,t)

Then P(x,t)dx is the probability of finding particle between x and x+dx at time.

$$\int_{-\infty}^{\infty} p(x,t)dx = 1$$

$$\int_{-\infty}^{\infty} \Psi * (x,t)\Psi(x,t) = 1$$

Then its normalized function

Orthogonally of wave function

1-Two different wave function Ψn,Ψm are said to be orthogonal if

$$\int_{-\infty}^{\infty} \Psi m * \Psi n d\tau = 0$$

2-Wave functions that are solution of given sch.equ. are usually orthogonal one to other.

3-Wave functions that are both orthogonal and normalized called (orthonormal)such that:

$$\int_{-\infty}^{\infty} \Psi m * \Psi n d\tau = \delta mn$$

$$\delta mn = 1 \; at \; m = n \qquad orthonormal$$

$$\delta mn = 0 \; at \; m \neq n$$

Expectation value

If the system is in state Ψ Which is not an eigen state of a such Observable ,then it is not possible to say with certainly what measured value will be found for A. Therefore, one has to use the average value $\bar{\bar{A}}$ which called in Quantum expectation value of A.

It is defined as:

$$\bar{A} = \langle A \rangle = \frac{\Psi * \widehat{A\Psi} d\tau}{\Psi * \ \Psi d\tau}$$

For normalized

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$$\bar{A} = \langle A \rangle = \Psi * \widehat{A\Psi} d\tau$$

Example:

1. position

$$\langle x\rangle = \int \Psi * \hat{x}\ \Psi \mathrm{dx}$$

2. Momentum

$$\langle px\rangle = \int \Psi * \widehat{px}\ \Psi \mathrm{dx}$$

$$px = -i\hbar \frac{\partial}{\partial x}$$

$$\langle px\rangle = -i\hbar \int \Psi * \frac{\partial}{\partial x} \Psi \mathrm{dx}$$

3-Total energy

$$\langle E \rangle = \int \Psi * \hat{E}\, \Psi dt$$

$$\hat{E} = i\hbar \frac{\partial}{\partial t}$$

$$\langle E \rangle = i\hbar \int \Psi * \frac{\partial}{\partial x} \Psi \mathrm{dt}$$

4-Kinetic energy

$$\langle T \rangle = \int \Psi * \hat{T}\, \Psi dx$$

$$\mathrm{T} = \frac{\widehat{p2}}{2m} = -\frac{\hbar^2}{2m} \frac{\partial \hat{}2}{\partial x \hat{}2}$$

$$\langle T \rangle = -\frac{\hbar^2}{2m} \int \Psi * \frac{\partial \hat{}2}{\partial x \hat{}2} \Psi \mathrm{dx}$$