Lecture 1

Third Stage

Quantum Mechanics

By

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Operators

Any mathematical entity which act on a wave function and change it to another function.

Example:- $\hat{A} = x$ $\qquad \varphi_x = x^3$

$$\hat{A}\varphi_x = x.x^3 = x^4 = \emptyset$$

Observables	Cla. Mech. Rep	Qua. Mech. Rep.
1-position	x	$\hat{X}$
2-Momentum	$p_x = m\dot{x}$	$\widehat{p_x} = -i\hbar\frac{\partial}{\partial x}$
3- kinetic energy	$T = \frac{p^2}{2m}$	$\hat{T} = \frac{\widehat{p^2}}{2m} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$
4- total energy	$E=T+V_{(x)}$	$\hat{E} = i\hbar\frac{\partial}{\partial t}$
5- Hamilton	$H = \frac{p^2}{2m} + V_{(x)}$	$\widehat{H} = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_{(x)}$

Eigen functions and Eigen values

When an operators on a function the outcome is another functions.

Example:- by using the eigen value equation show that the function $\varphi_n(x) = e^{i4x}$ is an eigen function of the operator $\hat{A} = \frac{\partial}{\partial x}$

$\hat{A} = \frac{\partial}{\partial x}$ $\qquad \varphi_n(x) = e^{i4x}$

$\hat{A}\varphi_n(x) = a_n\varphi_n(x)$

$\hat{A}\varphi_n(x) = \frac{\partial}{\partial}(e^{i4x})$

$=i4e^{i4x}$

$=a_n\varphi_n(x)$

$a_n = i4 \; eigen \; value$

$\varphi_n(x) = e^{i4x} eigenfunction$

By using the eigen value equation show that the function $\varphi_n(x) = \sin(6x)$ is an eigen function of the operator $\hat{A} = \frac{\partial^2}{\partial x^2}$ H.W.

Properties of operators

1-liner operator

$$i)\ \hat{A}(\varphi_1(x) + \varphi_2(x)) = \hat{A}\varphi_1 + \hat{A}\varphi_2$$

$$ii)\hat{A}\left(a\varphi_{(x)}\right) = a\hat{A}\varphi_{(x)}$$

2- Commutation

$$\hat{C} = [\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

$$i)\ if\ \hat{C} = 0 \rightarrow [\hat{A}, \hat{B}] = 0$$

$$then \quad \hat{A}\hat{B} = \hat{B}\hat{A} \qquad commutation$$

$$ii)\ if\ \hat{C} = 1$$

$$\hat{C} = Unit\ operator$$

$$iii)\ if\ \hat{C} \neq 0 \rightarrow [\hat{A}, \hat{B}] \neq 0$$

$$\hat{A}\hat{B} - \hat{B}\hat{A} \neq 0$$

$$then \quad \hat{A}\hat{B} \neq \hat{B}\hat{A} \quad not\ commutation$$

Example:- prove that $\left[\frac{\partial}{\partial x}, x\right]$ ***is unit operator***

$$\hat{C} = [\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

$$\hat{C} = \frac{\partial}{\partial x}x - x\frac{\partial}{\partial x} \qquad multiplyed \ \ \varphi_x$$

$$\hat{C}\varphi_x = [\frac{\partial}{\partial x}x - x\frac{\partial}{\partial x}]\varphi_x$$

$$\hat{C}\varphi_x = \left[\frac{\partial}{\partial x}x(\varphi_x) - x\frac{\partial}{\partial x}(\varphi_x)\right]$$

$$= \varphi_{(x)}\frac{\partial x}{\partial x} + x\frac{\partial \varphi_{(x)}}{\partial x} - \ x\frac{\partial \varphi_{(x)}}{\partial x}$$

$$\hat{c}\varphi_x = \varphi_x$$

Then C=$\frac{\varphi_x}{\varphi_x}$ then C =1

*****show that*** $[\hat{x}, \widehat{p_x}] = i\hbar$ ***H,W.***

Hint $p_x = i\hbar\frac{\partial}{\partial x}$

Hermitean operator

Operator $\hat{A}$ is said to be hermition when satisfying the relation:

$$\int_{-\infty}^{\infty} \varphi_n^* \hat{A}\varphi_m d\tau = \int_{-\infty}^{\infty} \varphi_m (\hat{A}\varphi_n)^* d\tau$$

Properties of hermitean operator

1) the eigen value correspond to any hermitean operator are real quantities.

2) eigen function correspond to different eigen value are always orthogonal i.e.

$$\int_{-\infty}^{\infty} \varphi_n^* \varphi_m d\tau = 0$$

Example prove that P_x is hermitian operator ?

$$\hat{A} = \widehat{p_x}$$

$$\int_{-\infty}^{\infty} \varphi_n^* \hat{A} \varphi_m d\tau = \int_{-\infty}^{\infty} \varphi_m \left(\hat{A}\varphi_n\right)^* d\tau$$

$$\int_{-\infty}^{\infty} \varphi_n^* (-i\hbar \frac{\partial}{\partial x}) \varphi_m dx$$

$$-i\hbar \int_{-\infty}^{\infty} \varphi_n^* \frac{\partial}{\partial x} \varphi_m dx ---------1$$

$$\int_{-\infty}^{\infty} u dv = uv_{-\infty}^{\infty}| - \int_{-\infty}^{\infty} v du$$

$$u = \varphi_n^* \qquad dv = \frac{\partial}{\partial x} \varphi_m dx$$

$$du = \frac{\partial \varphi_n^*}{\partial x} dx \qquad v = \varphi_m$$

$$= -i\hbar \varphi_n^* \varphi_m \Big|_{-\infty}^{\infty} + i\hbar \int_{-\infty}^{\infty} \varphi_m \frac{\partial \varphi_n^*}{\partial x} dx$$

$$= i\hbar \int_{-\infty}^{\infty} \varphi_m \frac{\partial \varphi_n^*}{\partial x} dx$$

$$= \int_{-\infty}^{\infty} \varphi_m \left(i\hbar \frac{\partial \varphi_n^*}{\partial x}\right) dx$$

$$= \int_{-\infty}^{\infty} \varphi_m \left(-i\hbar \frac{\partial \varphi_n}{\partial x}\right)^* dx$$

$$= \int_{-\infty}^{\infty} \varphi_m (p_x \varphi_n)^* dx$$

[P_x] is hermitian operator