Lecture 7

Third stage

Compton scattering

By

Eman Ahmed Abdulmunem Abdulkadhim

2) **Compton scattering**

In physics, Compton scattering is a type of scattering that X-rays and gamma rays undergo in matter. The inelastic scattering of photons in matter results in a decrease in energy (increase in wavelength) of an X-ray or gamma ray photon, called the Compton Effect. Part of the energy of the X/gamma ray is transferred to a scattering electron, which recoils and is ejected from its atom, and the rest of the energy is taken by the scattered photon.

The effect is important because it demonstrates that light cannot be explained purely as a wave phenomenon. Thomson scattering, the classical theory of an electromagnetic wave scattered by charged particles, cannot explain low intensity shifts in wavelength. Light must behave as if it consists of particles in order to explain the low-intensity Compton scattering. Compton's experiment convinced physicists that light can behave as a stream of particle-like objects (quanta) whose energy is proportional to the frequency.

Total energy before collision is = $E_T = h\nu + m_0c^2$ ....(3.1)

Total momentum before collision = $p_T = \frac{h\nu}{c}$ ....(3.2)

After collision:

Energy of recoil electron = $E_1^{/} = mc^2$ ....(3.3) m is the relativistic mass.

Momentum of recoil electron = $p_1^{/} = mv$ ....(3.4)

$P_1^{/}$ is in a direction making an angle ϕ with the direction of incident photon

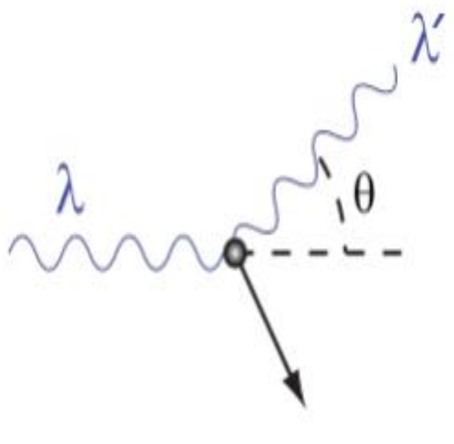
Energy of scattered photon = $E^{/} = h\nu^{/}$ ....(3.5)

Momentum of scattered photon = $p^{/} = \frac{h\nu^{/}}{c}$ ....(3.6)

$p^{/}$ is in a direction making an angle θ with the direction of incident photon.

The interaction between electrons and high energy photons (comparable to the rest energy of the electron, 511 keV) results in the electron being given part of the energy (making it recoil), and a photon containing the remaining energy being emitted in a different direction from the original, so that the overall momentum of the system is conserved. If the photon still has enough energy left, the process may be repeated..

If the photon is of lower energy, but still has sufficient energy (in general a few eV to a few KeV, corresponding to visible light through soft X-rays), it can eject an electron from its host atom entirely (a process known as the photoelectric effect), instead of undergoing Compton scattering. Higher energy photons (1.022 MeV and above) may be able to bombard the nucleus and cause an electron and a positron to be formed, a process called pair production.



A photon of wavelength λ comes in from the left, collides with a target at rest, and a new photon of wavelength λ' emerges at an angle θ.

By the early 20th century, research into the interaction of X-rays with matter was well underway. It was known that when a beam of X-rays is directed at an atom, an electron is ejected and is scattered through an angle θ. Classical electromagnetism predicts that the wavelength of scattered rays should be equal to the initial wavelength; however, multiple experiments found that the wavelength of the scattered rays was greater than the initial wavelength.

In 1923, Compton published a paper in the *Physical Review* explaining the phenomenon. Using the notion of quantized radiation and the dynamics of special relativity, Compton derived the relationship between the shift in wavelength and the scattering angle:

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta),$$

λ is the initial wavelength, λ' is the wavelength after scattering, h is the Planck constant, m_e is the mass of the electron, c is the speed of light, and θ is the scattering angle.

The quantity $\frac{h}{m_e c}$ is known as the Compton wavelength of the electron; it is equal to 2.43×10^{-12} m. The wavelength shift $\lambda' - \lambda$ is at least zero (for $\theta = 0°$) and at most twice the Compton wavelength of the electron (for $\theta = 180°$).

Compton found that some X-rays experienced no wavelength shift despite being scattered through large angles; in each of these cases the photon failed to eject an electron. Thus the magnitude of the shift is related not to the Compton wavelength of the electron, but to the Compton wavelength of the entire atom, which can be upwards of 10 000 times smaller.

A photon γ with wavelength λ is directed at an electron e in an atom, which is at rest. The collision causes the electron to recoil, and a new photon γ' with wavelength λ' emerges at angle θ. Let e' denote the electron after the collision.

From the conservation of energy,

$$E_\gamma + E_e = E_{\gamma'} + E_{e'}.$$

Compton postulated that photons carry momentum; thus from the conservation of momentum, the momentum of the particles should be related by

$$\mathbf{p}_\gamma = \mathbf{p}_{\gamma'} + \mathbf{p}_{e'},$$

Assuming the initial momentum of the electron is zero.

The photon energies are related to the frequencies by

$$E_\gamma = hf$$

$$E_{\gamma'} = hf'$$

where h is the Planck constant. From the relativistic energy-momentum relation, the electron energies are

$$E_e = m_e c^2$$

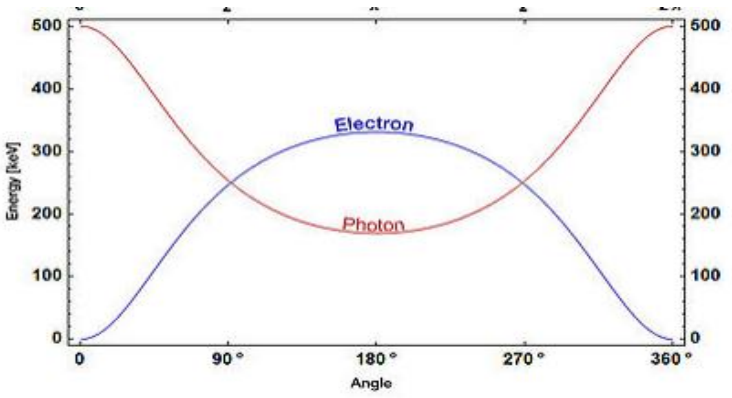
$$E_{e'} = \sqrt{(p_{e'}c)^2 + (m_e c^2)^2}.$$

Along with the conservation of energy, these relations imply that

$$hf + m_e c^2 = hf' + \sqrt{(p_{e'}c)^2 + (m_e c^2)^2}.$$

Then

$$p_{e'}^2 c^2 = (hf + m_e c^2 - hf')^2 - m_e^2 c^4. \qquad (1)$$



Energies of a photon at 500 keV and an electron after Compton scattering.

From the conservation of momentum,

$$\mathbf{p}_{e'} = \mathbf{p}_{\gamma} - \mathbf{p}_{\gamma'}.$$

Then by making use of the scalar product,

$$\begin{aligned} p_{e'}^2 &= \mathbf{p}_{e'} \cdot \mathbf{p}_{e'} = (\mathbf{p}_{\gamma} - \mathbf{p}_{\gamma'}) \cdot (\mathbf{p}_{\gamma} - \mathbf{p}_{\gamma'}) \\ &= p_{\gamma}^2 + p_{\gamma'}^2 - 2p_{\gamma}\, p_{\gamma'} \cos\theta. \end{aligned}$$

$$p_{e'}^2 c^2 = p_{\gamma}^2 c^2 + p_{\gamma'}^2 c^2 - 2c^2 p_{\gamma}\, p_{\gamma'} \cos\theta.$$

The relation between the frequency and the momentum of a photon is $pc = hf$, so

$$p_{e'}^2 c^2 = (hf)^2 + (hf')^2 - 2(hf)(hf')\cos\theta. \qquad (2)$$

Now equating 1 and 2,

$$(hf + m_e c^2 - hf')^2 - m_e^2 c^4 = (hf)^2 + (hf')^2 - 2h^2 f f' \cos\theta.$$

$$2hf m_e c^2 - 2hf' m_e c^2 = 2h^2 f f' \left(1 - \cos\theta\right).$$

Then dividing both sides by $2hff'm_e c$,

$$\frac{c}{f'} - \frac{c}{f} = \frac{h}{m_e c}\left(1 - \cos\theta\right).$$

Since $f\lambda = f'\lambda' = c$,

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta).$$