

Lecture 2
Third stage



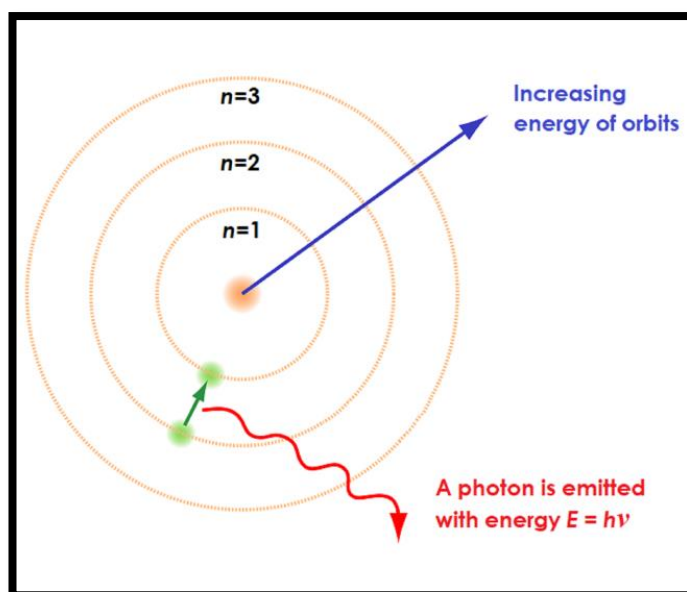
Quantum Mechanics

By

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Introduction to Quantum Theory:

Before we dive into the Schrodinger equation and how it contributed to the quantum model of waves as well as quantum theory, it would be best to learn what exactly quantum theory and the history behind it. Going back to the 1900s, Max Planck hypothesized that all energy is quantized and that light exhibits wave-particle duality. By saying something like energy can be “quantized”, it is simply referring to the existence of electrons on specific energy levels. More importantly, this means that energy can only be absorbed and emitted with specific values, and not just any possible value. The quantum model below shows these quantum levels that exist at the subatomic level.



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Planck also came up with an equation that is still constantly used today in modern physics called the Planck postulate:

$$E = h\nu \text{ (or) } E = hf$$

This equation restates that “photon energy (E) is proportional to its frequency (ν/f) multiplied by Planck’s constant (h) which is equal to 6.626×10^{-34} Js” (Weisstein 2).

This is important because if this quantum energy has a frequency, it also means that it must have a wavelength:

$$E = hf$$

$$f = \frac{c}{\Lambda}$$

$$E = \frac{hc}{\Lambda}$$

This postulation of wave-duality for all particles was hypothesized by the 20th century French physicist Louis de Broglie. De Broglie extended the theorem by stating all matter also possessed wave-duality attributes, meaning that all matter in the world possesses a wavelength of sorts. The reason humans don’t see wave-like characteristics in everyday objects in the world is due to inverse relationship between wavelength (Λ) and mass (m).

$$E = mc^2$$

$$E = hc/\Lambda$$

Setting these 2 formulas equal to each other and solving for Λ :

$$mc^2 = hc/\Lambda$$

$$\Lambda = h/mc$$

The Schrodinger Equation

Similar in classical mechanics, this formula is used to predict certain physical outcomes in a given system:

As well as another formula that is used to determine the energy for an object in a given system:

$$E_{\text{total}} = E_{\text{kinetic}} + E_{\text{potential}}$$

While the Schrodinger equation is written in different ways for certain reason, this form is consistent with the conservation of energy and is considered the “F=ma” and of quantum models:

$$E\Psi(x) = \frac{-\hbar^2}{2m} \frac{d^2\Psi(x)}{dx^2} + V\Psi(x)$$

The V in $V\Psi(x)$ is sometimes written as the potential energy of a function. The Ψ is the Greek symbol *psi* and known as the wave function, “which describes the probability of the quantum state of a particle in a system” (Cresser 171). So, if

$V\Psi(x)$ is the potential energy of an electron, then that means $= \frac{-\hbar^2}{2m} \frac{d^2\Psi(x)}{dx^2}$ must be the kinetic energy of the system. Deriving the formula for kinetic energy from the classical physics energy formula will give us this result:

$$E_t = E_k + E_p$$

$$E_t = \frac{1}{2}mv^2 + V$$

For substituting momentum (p) in the equation:

$$p = mv$$

$$E = \frac{p^2}{2m} + V$$

The general form of the wave equation where e is a mathematical constant, k is the wave vector, ω is the angular frequency (since this is a three-dimensional plane/system):

Differentiating the equation gives us:

$$\frac{d\Psi}{dx} = ike^{i(kx-\omega t)} = ik\Psi$$

We differentiate here because the derivative of the wave function can help describe the change in the state of the system as it evolves over time.

$$\frac{d^2\Psi}{dx^2} = i^2k^2e^{i(kx-\omega t)} = -k^2\Psi$$

The De Broglie relationship, where \hbar is the reduced Planck's constant, is:

$$k = \frac{p}{\hbar}$$

And by substituting k into $-k^2\Psi$:

$$\frac{d^2\Psi}{dx^2} = \frac{-p^2}{\hbar^2}\Psi$$

Multiplying \hbar^2 on both sides:

$$-\hbar \frac{d^2\Psi}{dx^2} = p^2\Psi$$

Now multiplying Ψ into the energy formula where $E = \frac{p^2}{2m} + V$, p^2 will turn into $p^2\Psi$

and then setting that term equal to $-\hbar \frac{d^2\Psi}{dx^2}$:

$$E\Psi = \frac{p^2\Psi}{2m} + V\Psi$$

$$E\Psi = \frac{-\hbar^2}{2m} \frac{d^2\Psi(x)}{dx^2} + V\Psi$$

Interpreting the Schrodinger Equation

The Schrodinger equation is not easy to interpret, but by breaking it down by comparing it to the classical physics energy equation, it might help us understand

$$E_t = E_k + E_p$$

$$E\Psi = \frac{-\hbar^2}{2m} \frac{d^2\Psi(x)}{dx^2} + V\Psi$$

I put the 2 equations on top of each other to help with picturing the distinct similarities they have. Notice how both equations solve for the total energy of something, that something being in this case a particle in space. Also take note of how both equations have 2 terms that add up to the total energy. Both of the 2 terms in the equations parallel to kinetic and potential energy of a system.

The wave function Ψ describes the probability of the quantum state of a particle in a system. In a 1-dimensional spinless particle, its state in the system is described by the wave function as:

$$\Psi(x,t)$$

Where x is the position and t is the time. Since the wave function only describes the probability of the position of the particle, it has all possibilities of being anywhere in the system at a given time. The probability density is another way of describing the relative likelihood that a value at any given sample in a space will be there. Similarly, in quantum mechanics, this function will give the probability amplitude of the particle from the wave function where $p(x)$ is a probability density function:

$$|\Psi(x,t)|^2 = \Psi(x,t)^* \Psi(x,t) = p(x,t)$$

The probability that this particle will be at position x in the interval $a \leq x \leq b$ is in the integral of the density:

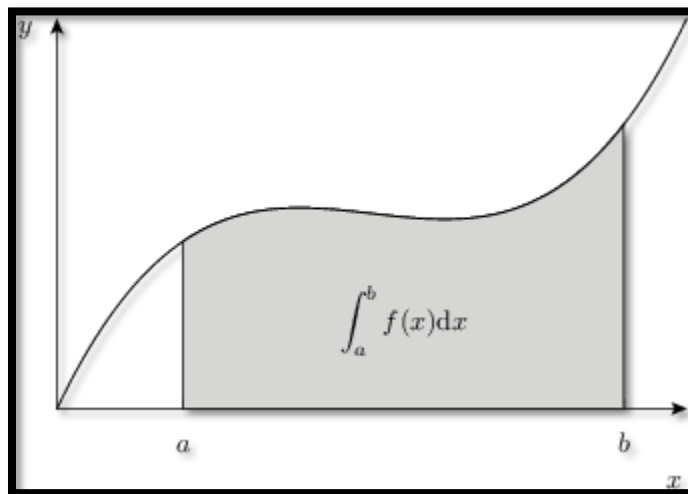
$$P_{a \leq x \leq b}(t) = \int_a^b dx |\Psi(x,t)|^2$$

A probability is a real number between 0 and 1, 0 meaning it has a 0% chance of happening and 1 meaning it has a 100% chance of happening. Since there is a 100% that if the particle is observed and measured, it will be always be somewhere.

This also means that a measurement of x must give a value between $-\infty$ and $+\infty$, and

$P_{-\infty < x < \infty}$ must equal 1:

$$\int_{-\infty}^{\infty} dx |\Psi(x, t)|^2 = 1$$



Take this graph to be a 1-dimensional space for example. By taking the integral of the equation, we can therefore say that there is a 100% chance that a particle will be found in the area under the graph (space) from intervals $a \leq x \leq b$.