

**Lecture 4**

**Third stage**



# ***Variance***

**By**

***Eman Ahmed Abdulmunem Abdulkadhim***

## *Variance*

The deviation in the measured of the operator A from its expected value  $\langle A \rangle$ .

$$\overline{(\nabla A)^2} = \langle A^2 \rangle - \langle A \rangle^2$$

## *Example:*

### 1. position

$$\overline{(Ax^2)} = \langle x^2 \rangle - \langle x \rangle^2$$

$$\langle x^2 \rangle = \int \Psi^* x^2 \Psi dx$$

$$\langle x^2 \rangle = \left( \int \Psi^* x^2 \Psi dx \right)^2$$

### 2. Momentum

$$\overline{(\nabla px)^2} = \langle px^2 \rangle - \langle px \rangle^2$$

$$\langle px^2 \rangle = \int \Psi^* P x \Psi dx$$

$$\langle px^2 \rangle = \left( \int \Psi^* P x \Psi dx \right)^2$$

## *Quantized states*

Some dynamic quantities, such as energy and angular momentum, take for themselves precisely distinct knowledge values and are said to be

quantized. This behavior is impossible in most cases, because applying the principles of quantum mechanics to a specific system often gives a distribution of these values that we obtain as a result of measurements (expected values). If there is a type of wave functions corresponding to the case in which a dynamic quantity is fully knowing the value, these functions full the following equation:

$$A\Psi=a\Psi$$

Where  $a$  is a real constant then the quantity  $A$  is fully defined by the quantity  $a$ . To demonstrate this, we calculate the expected value:

$$\begin{aligned}\langle A \rangle &= \int \Psi^* A \Psi d\tau \\ &= \int \Psi^* a \Psi d\tau \\ &= a \int \Psi^* \Psi d\tau \\ &= \langle A \rangle = a\end{aligned}$$

Likewise, the:

$$\begin{aligned}\langle A^2 \rangle &= \int \Psi^* A(A\Psi) d\tau \\ &= \int \Psi^* A a \Psi d\tau\end{aligned}$$

$$= a \int \Psi^* A \Psi d\tau$$

$$= a \int \Psi^* a \Psi d\tau$$

$$= a^2 \int \Psi^* \Psi d\tau$$

$$\therefore \langle A^2 \rangle = a^2$$

$$(\nabla A)^2 = \langle A^2 \rangle - \langle A \rangle^2$$

$$= a^2 - a^2 = 0$$

In other words, there is no doubt or truth about the value of A, but rather that A is defined by the exact value a.

### ***Parity***

Wave functions are often characterized by an important role that plays a fundamental role in physical phenomena, according to which the functions have an even and odd symmetry with respect to the reflection in coordinates at the point of origin.

$$\Psi(x) = \begin{cases} \Psi(-x) & \text{even parity} \\ \Psi(x) & \text{odd parity} \end{cases}$$

**Example 1:**

$$y(x) = \sin x$$

$$y(-x) = \sin (-x)$$

$$y(-x) = -\sin (-x) = -y(x)$$

**Example 2:**

$$y(x) = \cos x$$

$$y(-x) = \cos (-x)$$

$$y(-x) = \cos (x) = y(x)$$

If  $v(x)$  is symmetric, that is,  $v(-x) = v(x)$  and the energy levels are not dissolved, then all solutions to the time-dependent Schrödinger equation take the two forms

$$\Psi(-x) = \Psi(x) \quad \text{As for}$$

$$\Psi(-x) = -\Psi(x) \quad \text{or}$$

Where as  $\Psi(x)$  the non-time dependent Schrödinger equation is

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(x) + v(x)\Psi(x) = E\Psi(x)$$

And when replacing  $x$  with  $-x$

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(-x) + v(x)\Psi(-x) = E\Psi(-x)$$

Since  $\Psi(-x)$ ,  $\Psi(x)$  achieve one differential equation, which is the Schrödinger equation, which is not dependent on time, it must be

$$\Psi(-x) = r_n \Psi(x) \dots \dots \dots *$$

Where  $r_n$  is constant and substituting  $x$  with  $-x$

$$\Psi(x) = r_n \Psi(-x)$$

And using the equation \*, we find that

$$\Psi(x) = r_n r_n \Psi(x)$$

$$\Psi(x) = r_n^2 \Psi(x) \longrightarrow r_n = +1, -1$$

Returning to the equation (\*) and substituting for  $r_n$  is

$$\Psi(-x) = \mp \Psi(x)$$

In the case where  $\Psi(-x) = \Psi(x)$  the function is said to be an even symmetry, while in the other case  $\Psi(-x) = -\Psi(x)$  it is said to have a odd symmetry.

We can now define the reflection operator which has an effect on the function  $x$ )  $\Psi$  is to replace  $x$  with  $-x$

$$\hat{R}\Psi(x) = \Psi(-x)$$

**Reflection operator** : is that operator when operate on such function convert it around the origin i.e.  $\hat{R} \Psi(x) = \Psi(-x)$

With a simple modulation, the equation (\*) can be written as follows

$$\hat{R} \Psi(x) = r_n \Psi(x) = \Psi(-x)$$

Where  $r_n$  represents the eigenvalues of symmetry, and if we are now influenced again by the operator  $\hat{R}$  on both sides of the equation above, it will be:

$$\hat{R} (\hat{R} \Psi(x)) = \hat{R} r_n \Psi(x)$$

$$= r_n \hat{R} \Psi(x)$$

$$= r_n^2 \Psi(x)$$

$$\hat{R} (\hat{R} \Psi(x)) = \hat{R} r_n \Psi(x)$$

$$= \Psi(x)$$

$$r_n^2 = 1 \rightarrow r_n = \pm 1$$

That is, the eigenvalues of symmetry are  $\pm 1$ , which means that the wave functions of symmetry are either even in  $x$  or odd, here  $r_n = +1$  is the eigenvalues of the operator, while  $\Psi(x)$  is the eigenvalue.

**Example :**

If the operator  $\hat{H}(x)$  the Hamiltonian operator is an even function in  $x$  i.e.  $\hat{H}(-x) = \hat{H}(x)$  then prove that it is exchanged with the operator  $\hat{R}$  (reflection effect).

**the proof**

$$\hat{H} \hat{R}(x) \Psi(x) = \hat{H}(-x) \Psi(-x)$$

$$= \hat{H}(x) \Psi(-x)$$

$$= \hat{H}(x) \hat{R} \Psi(x)$$

Accordingly,

$$\hat{H} \hat{R}(x) \Psi(x) = \hat{H}(x) \hat{R} \Psi(x)$$

$$\hat{H} \hat{R}(x) - \hat{H}(x) \hat{R} \Psi(x) = 0$$

**That is, the substitution  $C$  of the two operators  $\hat{H}$ , equals zero**

$$C = \hat{H} \hat{R} - \hat{H} \hat{R} = 0$$