Lecture 5

Third stage

Quantization of energy

By

Eman Ahmed Abdulmunem Abdulkadhim

Quantization of energy

The foundation of quantum mechanics was laid in 1990 with max Planck's discovery of the quantized nature of energy .when plank developed his formula for black body radiation he was forced to assume that the energy exchanged between a black body and its thermal(electromagnetic) radiation is not a continuous quantity but needs to be restricted to discrete values depending on the (angular-frequency) of the radiation. Planck's formula can explain { all features of the black body radiation and his finding is phrased in the following way:

> Proposition 1.1.1 energy is quantized and given in units of $\hbar\,\omega$. $E=\hbar\,\omega$

Here ω denotes the angular frequency $\omega=2v\pi$. We will drop the prefix "angular" in the following and only refer to it as the frequency. We will also bear in mind the connection to the wavelength λ given by $c=\lambda v$, where c is the speed of light, and to the period T given by $v=1/T$.

The fact that the energy is proportional to the frequency, rather than to the intensity of the wave. What we would expect from classical physics – is quite counterintuitive. The proportionality constant $\hbar$ is called Planck's constant:

$$\hbar = \frac{h}{2\pi} = 1{,}054 \times 10^{-34}\, Js = 6{,}582 \times 10^{-16}\, eVs \qquad (1.1)$$

$$h = 6{,}626 \times 10^{-34}\, Js = 4{,}136 \times 10^{-15}\, eVs \qquad (1.2)$$

black body radiation

a black body is by definition an object that completely absorbs all light (radiation) that falls on it. This property makes a black body a perfect source of thermal radiation. A very good realization of a black body is an oven with a small hole. All radiation that enters through the opening has a very small probability of leaving through it again.

Thus the radiation coming from the opening is just the thermal radiation, which will be measured in dependence of its frequency and the oven temperature. Such radiation sources are also referred to as (thermal) cavities. The classical law that describes such thermal radiation is the Rayleigh-jeans law which express the energy density u(w) in terms of frequency w and temperature T.

$$Theorem\ 1.1\ (Rayleigh - Jeans\ law)\ u(w) = \frac{kT}{\pi^2 c^3}\, w^2$$

Where k denotes Boltzmann's constant, $k = 1{,}38 \times 10^{-23}\, JK^{-1}$

Boltzmann's constant plays a role in classical thermo-statistics where (ideal) gases are analyzed whereas here we describe radiation. The quantity kT has the dimension of energy, in a classical system in thermal equilibrium, each degree of freedom (of motion) has a mean energy of $E = \frac{1}{2}\ KT - Equipartition\ Theorem.$

From the expression of theorem 1.1 we immediately see that the integral of the energy density over all possible frequency is divergent,

$$\int_0^\infty dw\ u(w) \rightarrow\ \infty \qquad (1.3)$$

Which would imply an infinite amount of energy in the black body radiation . This is known as the ultraviolet catastrophe and the law is therefore only valid for small frequencies.

For high frequencies a law was found empirically by Wilhelm Wien in 1896.

$$Theorem\ 1.2\ (Wiens\ law) \quad u(w) \rightarrow Aw^3 e^{-B\frac{w}{T}} \quad for \quad w \rightarrow \infty$$

Where A and B are constants we will specify later on.

As already mentioned Max Planck derived an impressive formula which interpolates between the Rayleigh – Jeans law and Wien's law. For the derivation the correctness of proposition 1.1.1 was necessary, the energy can only occur in quanta of ħ ω. For this achievement he was awarded the 1919 noble prize in physics.

$$Theorem\ 1.3\ (plancks\ law) \qquad u(w) = \frac{ħ}{\pi^2 c^3} \frac{w^3}{\exp\left(\frac{ħw}{kT} - 1\right)}$$

From Planck's law we arrive at the already well-known laws of Rayleigh –Jeans and Wien by taking the limits for $\omega \to 0\ or\ \omega \to \infty\ respectively.$

For $\omega \ \to 0$ $\qquad Rayleigh - Jeans$

$$u(w) = \frac{ħ}{\pi^2 c^3} \frac{w^3}{\exp\left(\frac{ħw}{kT}\right)} - 1$$

For $\omega \to \infty$ $\qquad \to Wien$

- **Wien's displacement law:** $\lambda_{max} T = const. = 0.29\ cm\ K$
- **Stefan-Boltzmann law**: for the radiative power we have

$$\int_0^\infty dw\, u(w) \propto T^4 \int_0^\infty d\left(\frac{ħ\omega}{kT}\right) \frac{\left(\frac{ħ\omega}{kT}\right)^3}{\exp\left(\frac{ħ\omega}{kT}\right) - 1} \qquad (1.4)$$

Substituting $\frac{ħ\omega}{kT} = x\ and\ using\ formula$

$$\int_0^\infty dx \frac{x^3}{e^x - 1} = \frac{\pi^4}{15} \qquad (1.5)$$

We find the proportionality $\propto T^4$

The photoelectric effect

Facts about the photoelectric

In 1887 Heinrich Hertz a discovered a phenomenon the photoelectric effect that touché the foundation of quantum mechanics. A metal surface emits electron when illuminated by ultraviolet light .the importance of this discovery lies within the inability of classical physics to describe the effect in its full extent based on three observations.

1- The kinetic energy of the emitted electron is independent of the intensity of the illuminating source of light.

2- Increases with increasing frequency of the light.

3- There exists a threshold frequency below which no electrons are emitted.

More accurate amassments were made by Philipp Lenard between 1900 and 1902 for which he received the Nobel prize in 1905. In terms of classical physics this e_ ect was not understood as from classical electrodynamics.

Was known that the

$$energy\ density: \qquad u = \frac{1}{8\pi}\left(\overrightarrow{E^2} + \overrightarrow{B^2}\right) \qquad (1.6)$$

And the

$$energy\ flux: \qquad \frac{c}{8\pi}\ \vec{E} \times \vec{B} \qquad (1.7)$$

Are both proportional to the intensity. Thus knocking electrons out of the metal is possible , but there is no threshold that limits this process to certain frequencies. As long as the surface is exposed to the radiation the electrons absorb energy until they get detached from the metal.

Einstein Explanation for the photoelectric effect

The phenomenon of the photoelectric effect could then be explained by albert Einstein in 1905 with his photon hypothesis which he was also awarded the Nobel prize for in 1921.

Einstein explained the effect in the following way. The incident photons transfer their energy to the electrons of the metal. Since the electrons are bound to the metal the energy need to be sufficient to overcome the electrostatic barrier of the metal. The respective energy, which is dependent on the material used is termed the work function of the material.

$$Proposition\ 1.2.2\ (Photoelectric\ formula)\quad E_{Kin} = \frac{mv^2}{2} = ħ\omega - W$$

Where W is the work function of the metal. We understand that the kinetic energy of the emitted electrons is bounded by the frequency of the incident light such that $E_{Kin.e} -< ħ\omega\ photon$ and we conclude that a threshold frequency w_0 must exist where the emitted electrons are at rest $E_{kin,e} = 0$ after escaping the potential of the metal.

$$\text{Threshold frequency } w_0 = \frac{W}{ħ}$$