Republic of Iraq

Ministry of Higher Education

and Scientific Research

Al-Mustaqbal University College

Computer Engineering Techniques Department

Subject: Fundamentals of Electrical Engineering

First Class

Lecture Eleven

By

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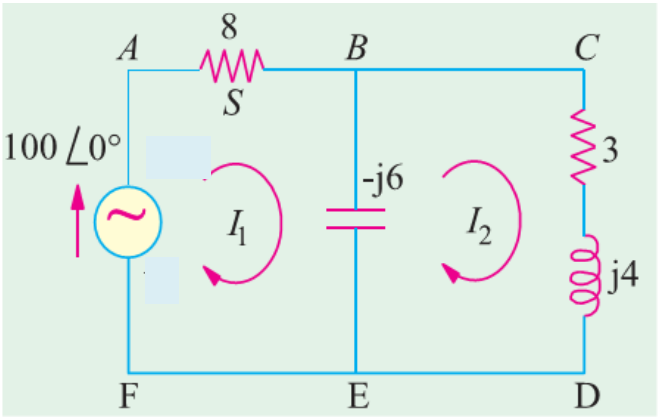
التحليل الحلقي (Mesh Analysis)

بالاعتماد على استخدام قانون كريشوف للفولتية يتم تطبيق طريقة التحليل الحلقي في دوائر التيار المستمر

Example 1. Find the power output of the voltage source in the circuit of Fig. below. Prove that this power equals the power in the circuit resistors.



Sol:

Loop 1

$$8I_1 - j6(I_1 - I_2) - 100\angle 0 = 0$$

$$(8 - j6)I_1 + j6I_2 = 100\angle 0$$

Loop 2

$$-j6(I_2 - I_1) + (3 + j4)I_2 = 0$$

$$j6I_1 + (3 - j2)I_2 = 0$$

$$\begin{bmatrix} (8-j6) & j6 \\ j6 & (3-j2) \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 100\angle 0^o \\ -0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} (8-j6) & j6 \\ j6 & (3-j2) \end{bmatrix} = (8-j6)\ (3-j2) - (j6)^2 :$$

$$= 24 - j16 - j18 - 12 + 36$$

$$= 48 - j\ 34 = 58.82\angle - 35.3^0$$

$$\Delta_1 = \begin{vmatrix} 100\angle 0^o & j6 \\ 0 & (3-j2) \end{vmatrix} = (300 - j\ 200) = 360\angle - 33.7$$

$$\Delta_2 = \begin{vmatrix} (8-j6) & 100\angle 0^o \\ j6 & 0 \end{vmatrix} = 600\ \angle 90^o$$

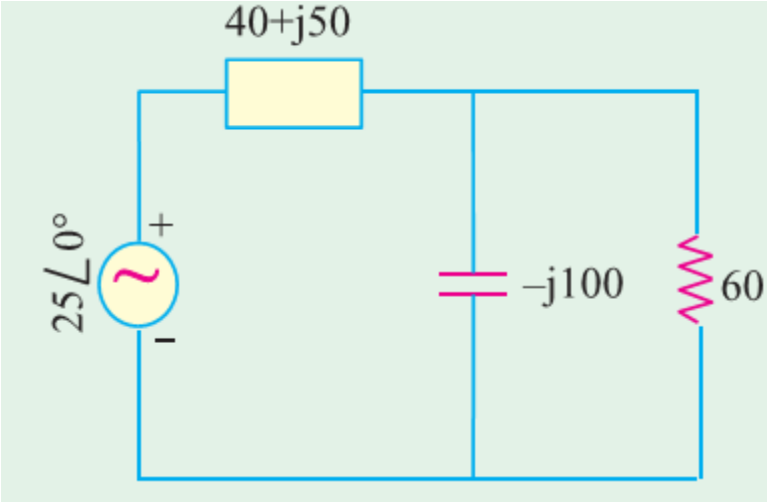
$$\boldsymbol{I_1} = \frac{\boldsymbol{\Delta_1}}{\Delta} = \frac{300 - j200}{48 - j34} \times \frac{48 + j34}{48 + j34}$$

$$= \frac{14400 + j10200 - j9600 + 6800}{2304 + 1156} = \frac{21200 + j600}{3460} = 6.13 + j0.17\ A$$

$$\boldsymbol{I_2} = \frac{\boldsymbol{\Delta_2}}{\Delta} = \frac{j600}{48 - j34} \times \frac{48 + j34}{48 + j34}$$

$$= \frac{j28800 - 20400}{2304 + 1156} = \frac{j28800 - 20400}{3460} = -5.89 + j8.32\ A$$

Example 2: Using mesh analysis method, find the value of current in each branch of the network shown in Fig. below



Sol:

Loop No. 1

$$25 - I_1 (40 + j50) - (-j\,100)(I_1 - I_2) = 0$$

$$\therefore \quad 25 - I_1 (40 - j50) - j\,100\, I_2 = 0$$

Loop No. 2

$$-60\, I_2 - (-j100)(I_2 - I_1) = 0$$

$$\therefore \quad -j100\, I_1 - I_2 (60 - j100) = 0$$

$$\therefore \qquad I_2 = \frac{-j100 I_1}{(60 - j100)} = \frac{100\angle -90^\circ I_1}{116.62\angle -59^\circ} = 0.8575\angle 31^\circ I_1$$

$$25 - I_1 (40 - j50) - j100 \times 0.8575\ \angle 31^\circ\ I_1 = 0$$

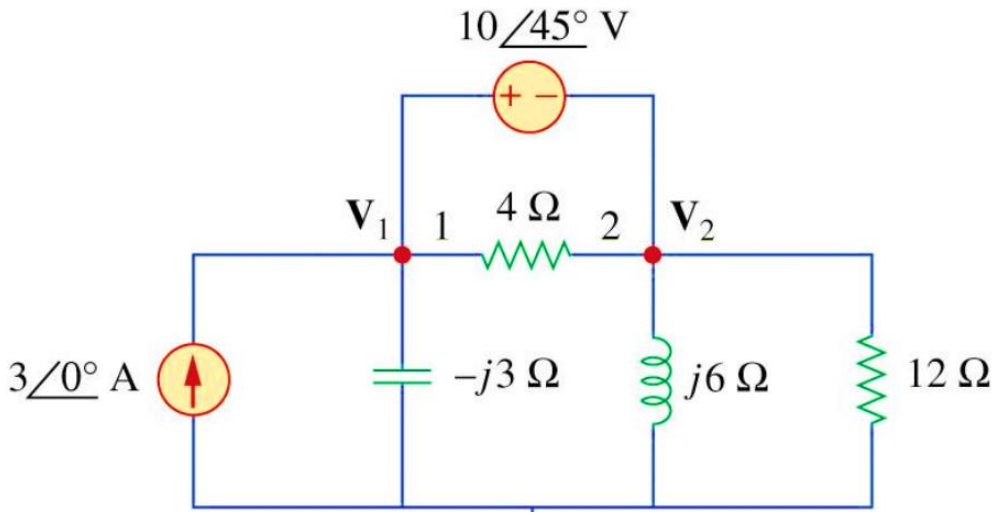
$$25 - 40\, I_1 + j50\, I_1 - 85.75\, \angle 59^\circ$$
$$25 - I_1\,(84.16 + j\,23.5) \;=\; 0.$$

$$I_1 = \frac{25}{(84.16 + j23.5)} = \frac{25}{87.38\,\angle 15.6^\circ}\; 0.286\, \angle -15.6^\circ \text{ A}$$

$$I_2 = 0.8575\, \angle 31^\circ\, I_1 \times 0.286\, \angle -15.6^\circ = 0.2452\, \angle 46.6^\circ \text{A}$$

Current through the capacitor = $(I_1 - I_2) = 0.286\, \angle 15.6^\circ - 0.2452\, \angle 46.6^\circ = 0.107 + j0.1013 = 0.1473\, \angle 43.43^\circ$ A.

Example 3: determine the value of current in each element using mesh analysis method.



Sol:

Loop 1

The value of current in first loop is

$$I_1 = 3 + j0\ A$$

Loop 2

$$-j3(I_2 - I_1) + 4(I_2 - I_4) + j6(I_2 - I_3) = 0$$

$$j3 \times 3 + (4 + j3)I_2 - j6\, I_3 - 4I_4 = 0$$

$$(4 + j3)I_2 - j6\, I_3 - 4I_4 = -j9 \qquad (1)$$

Loop 3

$$j6(I_3 - I_2) + 12\, I_3 = 0$$

$$-j6I_2 + (12 + j6)I_3 = 0 \qquad (2)$$

Loop 4

$$10\angle 45 = 10(\cos 45 + j \sin 45)$$

$$= 7.07 + j7.07$$

$$4(I_4 - I_2) + 10\angle 45 = 0$$

$$4I_2 - 4I_4 = -7.07 - j7.07 \qquad (3)$$

The current in the second loop is

$$I_2 = \frac{\begin{vmatrix} -j9 & -j6 & -4 \\ 0 & 12 + j6 & 0 \\ -7.07 - j7.07 & 0 & -4 \end{vmatrix}}{\begin{vmatrix} 4 + j3 & -j6 & -4 \\ -j6 & 12 + j6 & 0 \\ 4 & 0 & -4 \end{vmatrix}}$$

$$= \frac{-j9[(12 + j6) * -4] - (-j6)[0] + (-4)[0 - (-7.07 - j7.07) * (12 + j6)]}{(4 + j3)[(12 + j6) * -4] - (-j6)[-j6 * -4] - 4[0 - 4(12 + j6)]}$$

$$= \frac{46.32 - j77.04}{-456 - j336} = 0.015 + j0.158\ A$$

$$I_3 = \frac{\begin{vmatrix} 4+j3 & -j9 & -4 \\ -j6 & 0 & 0 \\ 4 & -7.07-j7.07 & -4 \end{vmatrix}}{\begin{vmatrix} 4+j3 & -j6 & -4 \\ -j6 & 12+j6 & 0 \\ 4 & 0 & -4 \end{vmatrix}}$$

$$= \frac{(4+j3)*0 - (-j9)[-j6*-4] - 4[-j6*(-7.07-j7.07)]}{(4+j3)[(12+j6)*-4] - (-j6)[-j6*-4] - 4[0-4(12+j6)]}$$

$$= \frac{-46.32 - j169.68}{-456 - j336} = 0.244 + j0.193\ A$$

$$I_4 = \frac{\begin{vmatrix} 4+j3 & -j6 & -j9 \\ -j6 & 12+j6 & 0 \\ 4 & 0 & -7.07-j7.07 \end{vmatrix}}{\begin{vmatrix} 4+j3 & -j6 & -4 \\ -j6 & 12+j6 & 0 \\ 4 & 0 & -4 \end{vmatrix}}$$

$$= \frac{(4+j3)[(12+j6)*(-7.07-j7.07)] - (-j6)[-j6*(-7.07-j7.07)] - j9[-4(12+j6)}{(4+j3)[(12+j6)*-4] - (-j6)[-j6*-4] - 4[0-4(12+j6)]}$$

$$= \frac{-258.42 - j458.82}{-456 - j336} = 0.848 + j0.381\ A$$