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## Analog Computer Components, Systems

### 1. Introduction:

The electronic machines tend to fall mainly into two broad classes, the “digital” machines and the “analogue” machines. In this section we presented the main features of electronic machines in such a way as to emphasize differences between the digital and analogue types.

The analogue machine is in no sense a replacement or substitute for the digital machine, and indeed it is rare to find problems which can be solved equally well by either type, and hence the present-day effort being put into the development of hybrid machines.

The two main types (digital and analog) have features which fit them for different fields of application, and which make them attractive in different degrees to mathematicians, physicists, engineers, accountants, and many other users.

### 2. Digital Machines

Digital machines handle quantities represented as integral numbers of electrical pulses. Hence, continuous changes in the values of variables cannot be represented exactly, because the number of pulses representing a quantity cannot change by less than a single pulse. Despite its various advantages, for certain classes of problems digital machines are much slower than analogue machines. Additionally, digital machines need a central set of equipment for pulse generating which does not alter rapidly in size as the capacity of the machine changes and its size cannot be reduced below a certain minimum. Hence the machine cannot be smaller than this central equipment allows.

### 3. Analog Machines

The common feature of analogue machines is that the various quantities in the problem to be solved are represented by corresponding physical quantities in the machine. In



analogue machines the analogue quantities are commonly voltages which correspond in some predetermined manner with the quantities in the problem.

Analogue machine need no central set of equipment corresponding to the pulse-generating equipment of the digital machine, and it is economical and practical to build quite small machines and extend these later if required.

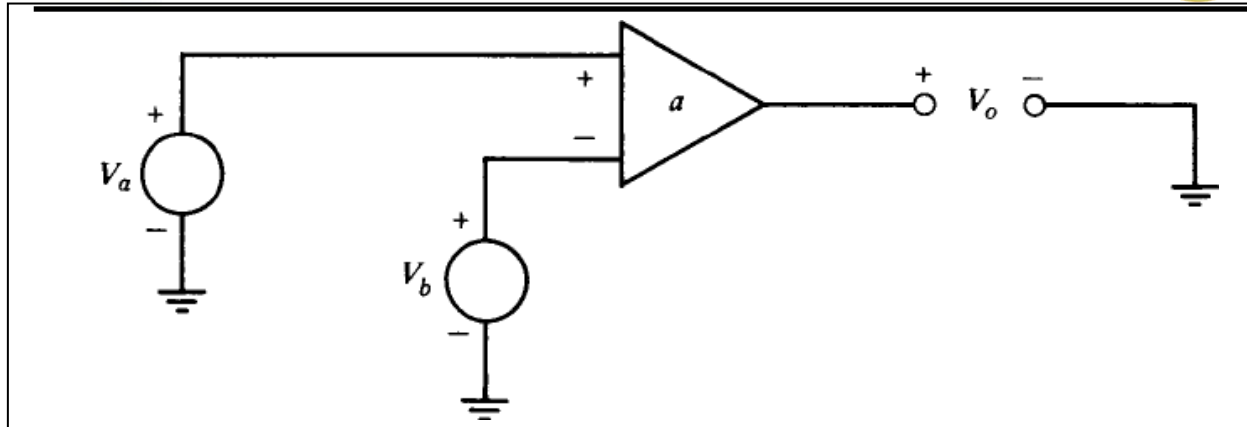
Electronic analogue machines can perform addition, subtraction and some other operations directly, and can deal with continuously varying quantities. In particular they can perform integration directly, provided that the independent variable is time.

Analogue machines have found widest application in the fields of aerodynamics and electrodynamics. When the analogue machine is used in this way the variables and constants in the machine, and the way these quantities react on each other, often present a close parallel with the behavior of the actual system being studied, so the machine is effectively a model of the system (simulator). Analogue machines of this type are now highly developed as flight simulators, unclear reactor control simulators, and missile training equipment.

#### **4. Operational Amplifiers (OP-AMP)**

The term operational amplifier evolved from original applications in analog computation where these circuits were used to perform various mathematical operations such as summation and integration. Because of the performance and economic advantages of available units, present applications extend far beyond the original ones, and modern operational amplifiers are used as general purpose analog data-processing elements.

The OP-AMP is a simple building block of analogue machines, it has two inputs, one is called the inverting input (often labeled -) and other is called the non-inverting input (often labeled +). And usually has single output. The OP-AMP also has two power supply connections, one for the positive rail and one for the negative rail.



- The implied relationship among input and out variables is:

$$V_{\text{out}} = a (V_{+} - V_{-}) \text{ Where } a \text{ is the open loop gain}$$

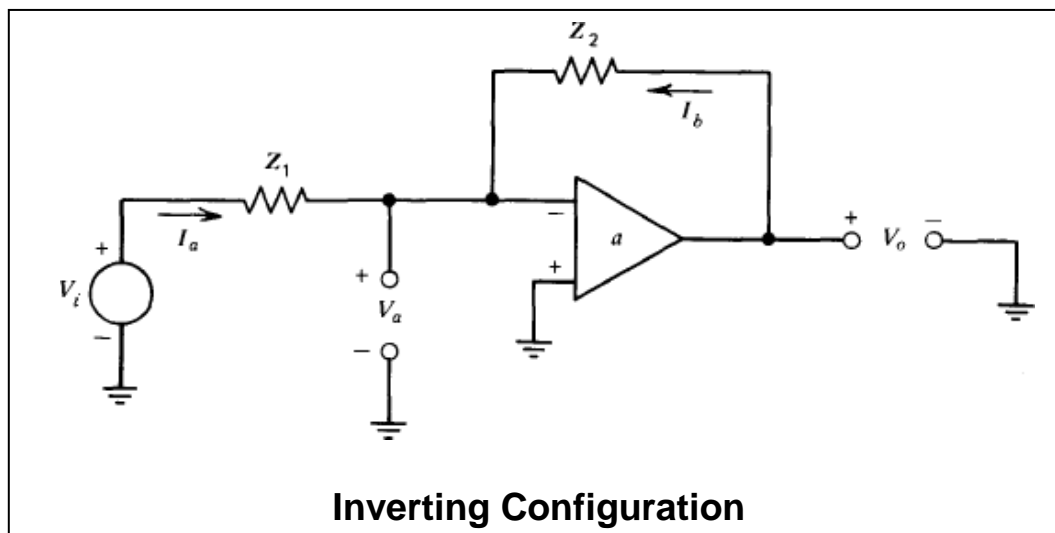
- The OP-AMP is a high gain DC amplifier; the output voltage is simply the difference in voltage between the inverting and non-inverting inputs, multiplied by the gain.
- The OP-AMP must have feedback in order to perform useful functions.

## 5. Feedback and the OP-AMP

There are two basic ways of applying feedback to an OP-AMP:

- The inverting configuration.
- The non-inverting configuration.

### 5.1. The inverting configuration:





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To calculate the closed-loop gain two conditions must satisfied.

1. The differential input voltage is zero.
2. No current flows into either input terminal of the ideal amplifier.

Let us look at the case of an inverting AMP in a little more detail. Referring to the above figure in more details, the non-inverting terminal is connected to ground. Since the OP AMP will force the differential voltage across the inputs to zero, the inverting input will also appear to be at ground. In fact, this node is often referred as “virtual ground”.

If there is a voltage ( $V_i$ ) applied to the input resistor ( $Z_1$ ), it will set up a current ( $I_a$ ) through resistor ( $Z_1$ ). Since the input impedance of the OP-AMP is finite, no current will flow into the inverting input, therefore the same current ( $I_b$ ) must flow through the feedback resistor ( $Z_2$ ). Hence :

$$I_a = I_b$$

$$\frac{V_i - V_a}{Z_1} = \frac{V_a - V_o}{Z_2}$$

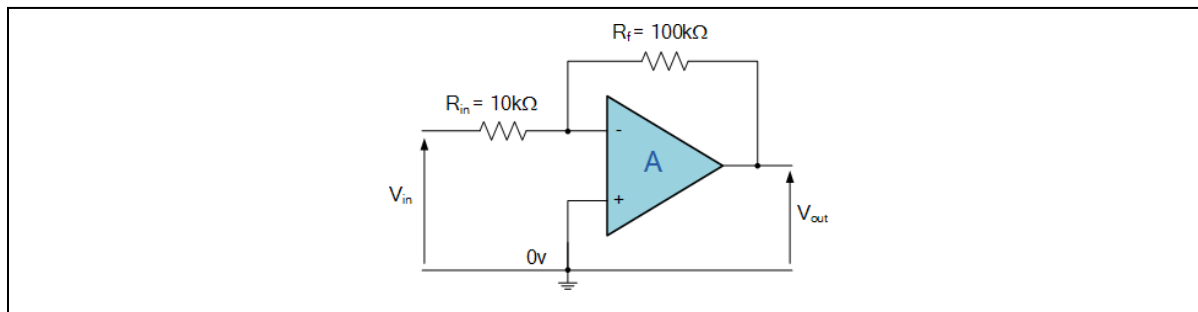
$$\frac{V_i}{Z_1} = \frac{-V_o}{Z_2}$$

$$\frac{V_o}{V_i} = \frac{-Z_2}{Z_1}$$

$$\text{Closed Loop Gain (A)} = \frac{-Z_2}{Z_1}$$



**Example No.1:** Find the closed loop gain of the following inverting amplifier circuit.



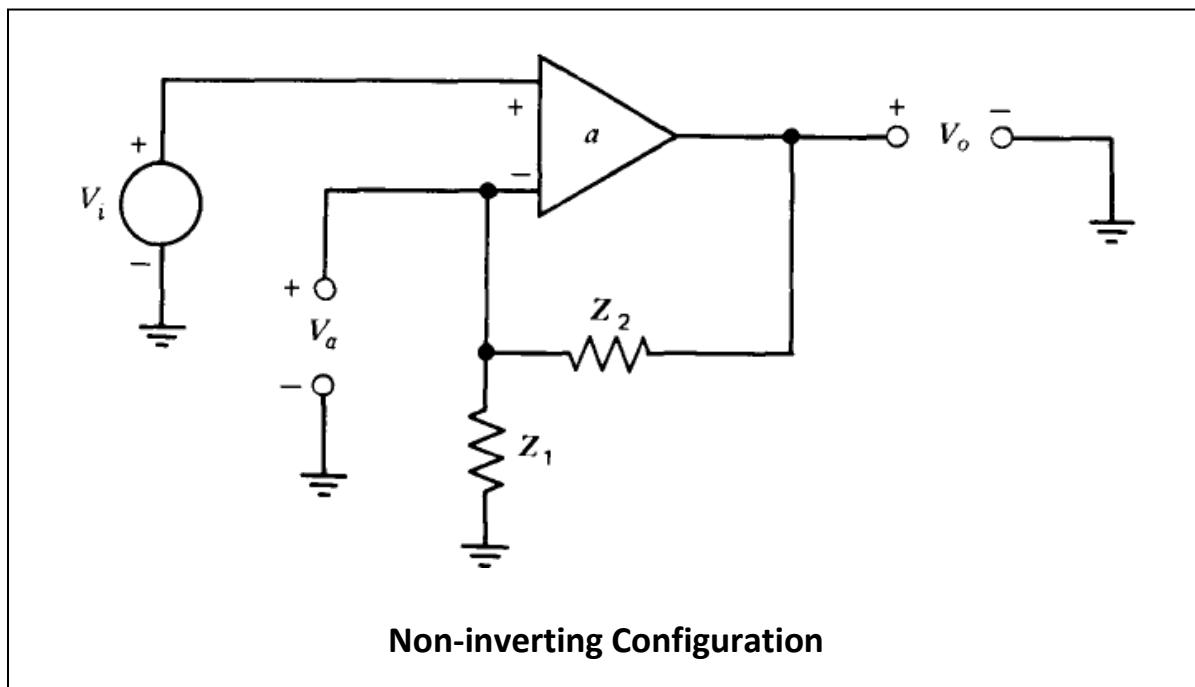
**Solution:**

$$\text{Closed Loop Gain (A)} = \frac{-Z_2}{Z_1}$$

$$(A) = -100\text{k}/10\text{k} = -10$$

**Example 2:** To increase the Gain of example (1) to 40, find the value of the resistor ( $R_2$ ) required? (Homework)

### 5.2. The Non-inverting configuration:





Let examine the above figure in more details, the input voltage is applied to the non-inverting terminal. The output voltage drives a voltage divider consisting of  $Z_1$  and  $Z_2$ . The voltage at the inverting terminal  $V_a$ , which is at the junction of the two resistors, is:

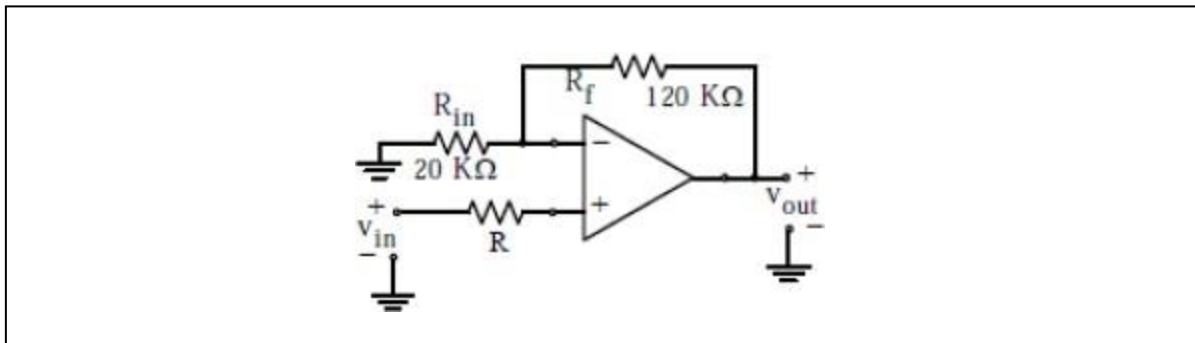
$$V_a = \frac{Z_1}{Z_1 + Z_2} V_o$$

Since  $V_i$  equal to  $V_a$

$$\frac{V_o}{V_i} = \frac{Z_1 + Z_2}{Z_1}$$

$$\text{The Close loop Gain (A)} = \frac{v_o}{v_i} = 1 + \frac{z_2}{z_1}$$

**Example:** compute the voltage gain and the output voltage for the non-inverting OP-AMP circuit shown in figure below, given that  $V_i = 1$  mV.



**Solution:**

$$\text{Voltage Gain (A)} = 1 + \frac{z_2}{z_1}$$

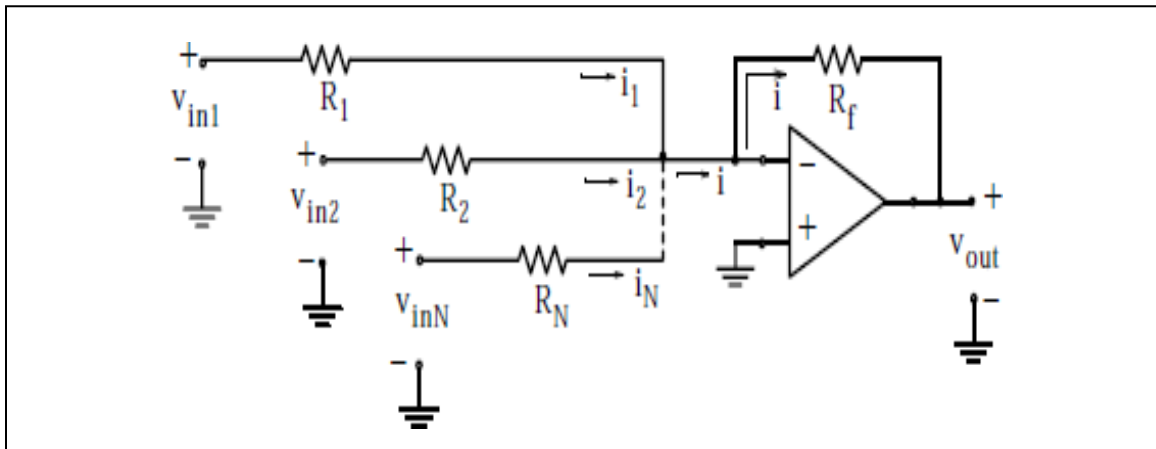
$$A = 1 + \frac{120 K\Omega}{60 K\Omega} = 1 + 6 = 7$$

$$V_{out} = A V_{in} = 7 * 1 \text{ mV} = 7 \text{ mV.}$$



## 6. The OP-AMP as Adder (Summing) and Averaging Circuit.

The figure below shows the basic inverting summing and averaging OP-AMP circuit.



In the above circuit, the total current is:

$$I = I_1 + I_2 + \dots + I_N$$

$$I_1 = \frac{V_{in1}}{R_1}, I_2 = \frac{V_{in2}}{R_2}, \dots, I_N = \frac{V_{inN}}{R_N}$$

$$V_{out} = - R_f I$$

$$V_{out} = - R_f (I_1 + I_2 + \dots + I_N)$$

$$V_{out} = - R_f \left( \frac{V_{in1}}{R_1} + \frac{V_{in2}}{R_2} + \dots + \frac{V_{inN}}{R_N} \right)$$

$$V_{out} = - \left( \frac{R_f}{R_1} V_{in1} + \frac{R_f}{R_2} V_{in2} + \dots + \frac{R_f}{R_N} V_{inN} \right)$$

If  $R_1 = R_2 = \dots = R_N = R$

$$V_{out} = - \frac{R_f}{R} (V_{in1} + V_{in2} + \dots + V_{inN})$$

If  $R_f = R$

$$V_{out} = - (V_{in1} + V_{in2} + \dots + V_{inN})$$

The above equation indicates that the circuit of the OP-AMP adder can be used to find the negative sum of any number of input voltages.

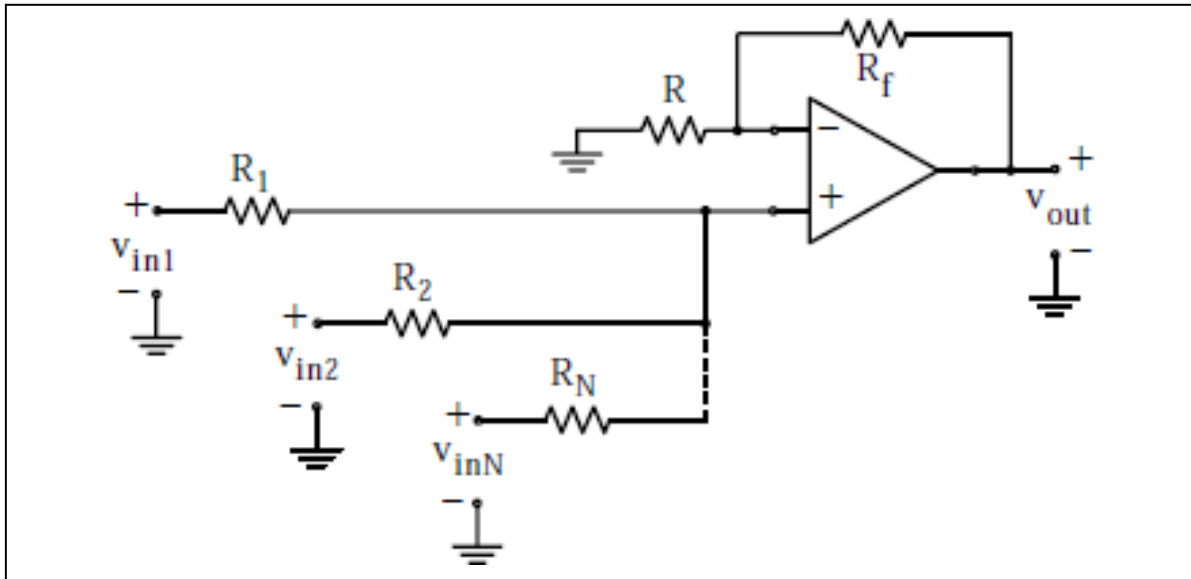
The above circuit can also be used to find the average value of all input voltages.

The ratio  $\frac{R_f}{R}$  is selected such that the sum of the input voltage is divided by the number of the input voltages applied at the inverting input of the OP-AMP. For example, if we have three inputs voltages, hence the ratio  $\frac{R_f}{R} = \frac{1}{n} = \frac{1}{3}$ , and overall

equation will be :  $V_{out} = - (V_{in1} + V_{in2} + V_{in3}) / 3$ .

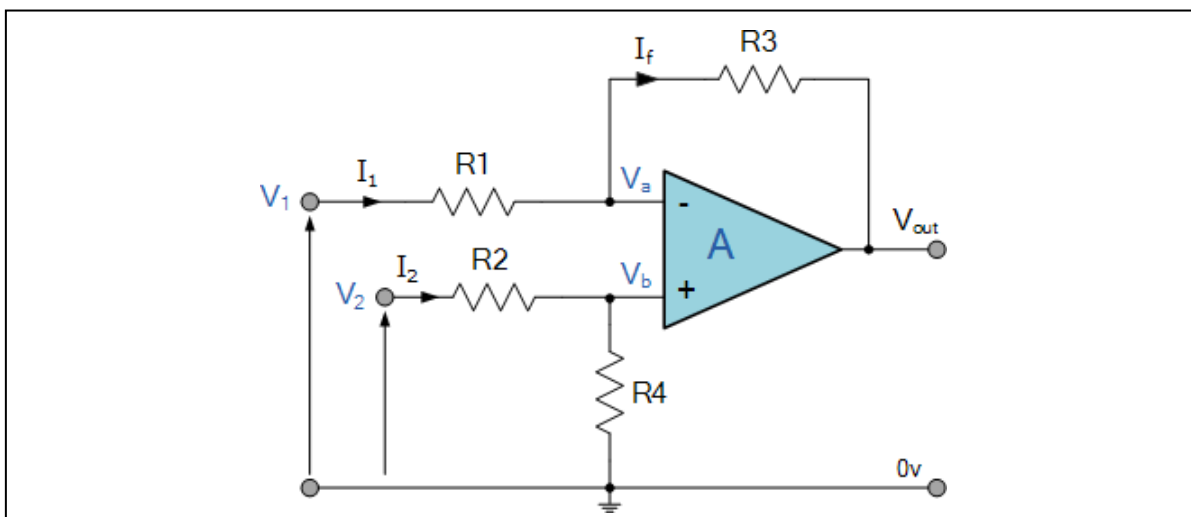


Figure below shows the non-inverting summing, averaging OP-AMP. It can be seen that the voltage sources  $V_{in1}$ ,  $V_{in2}$ , ...,  $V_{inN}$  and the associated resistors  $R_1$ ,  $R_2$ , .....,  $R_N$  can be replaced by the current sources whose values are:  $\frac{v_{in1}}{R_1}$ ,  $\frac{v_{in2}}{R_2}$ , .....,  $\frac{v_{inN}}{R_N}$  respectively.



## 7. The OP-AMP as Differential (subtraction) Circuit:

The subtraction OP-AMP amplifier amplifies the voltage difference present on its inverting and non-inverting inputs. Figure below shows the OA-AMP as difference (subtraction) circuit.







The differential amplifier circuit allows input signals to be applied simultaneously to both input terminals and produce an output of the difference between the input signals. However, to solve for  $V_{out}$  we will connect each input to ground in turn. Consequently, the rules of inverting and non-inverting circuits will be applied.

$$V_a = V_b$$

$$V_b = V_2 \frac{R_4}{R_2 + R_4}$$

When  $V_2 = 0$ , (inverting) then,  $V_{out(a)} = -V_1 \frac{R_3}{R_1 + R_3}$

when  $V_1 = 0$ , (Non - inverting) then  $V_{out(b)} = V_2 \frac{R_4}{R_2 + R_4} \left( 1 + \frac{R_1 + R_3}{R_1} \right)$

$$\text{When } V_a = V_b = V_2 \frac{R_4}{R_2 + R_4}$$

$$\text{then, } V_{out(b)} = V_2 \frac{R_4}{R_2 + R_4} \left( 1 + \frac{R_1 + R_3}{R_1} \right)$$

Since,  $V_{out} = V_{out(a)} + V_{out(b)}$

$$\text{then, } V_{out} = -V_1 \frac{R_3}{R_1 + R_3} + V_2 \frac{R_4}{R_2 + R_4} \left( 1 + \frac{R_1 + R_3}{R_1} \right)$$

Speical case (1): if  $R_1 = R_2 = R_3 = R_4$

$$\text{Then : } V_{out} = V_2 - V_1$$

Speical case (2): if  $R_1 = R_2$  and  $R_3 = R_4$

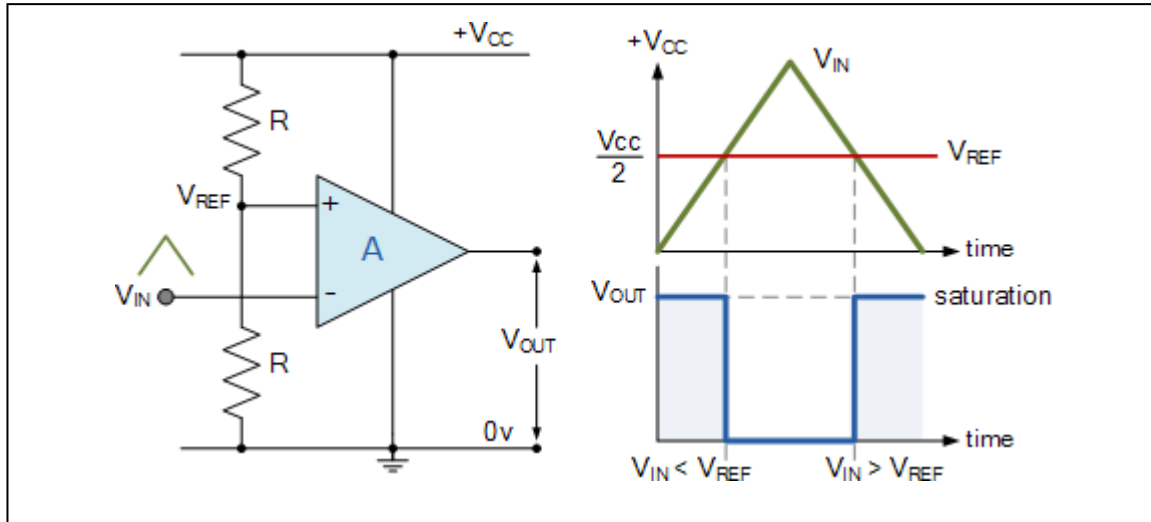
$$\text{Then, } V_{out} = \frac{R_3}{R_1} [V_2 - V_1]$$

## 8. The OP-AMP as Comparator Circuit:

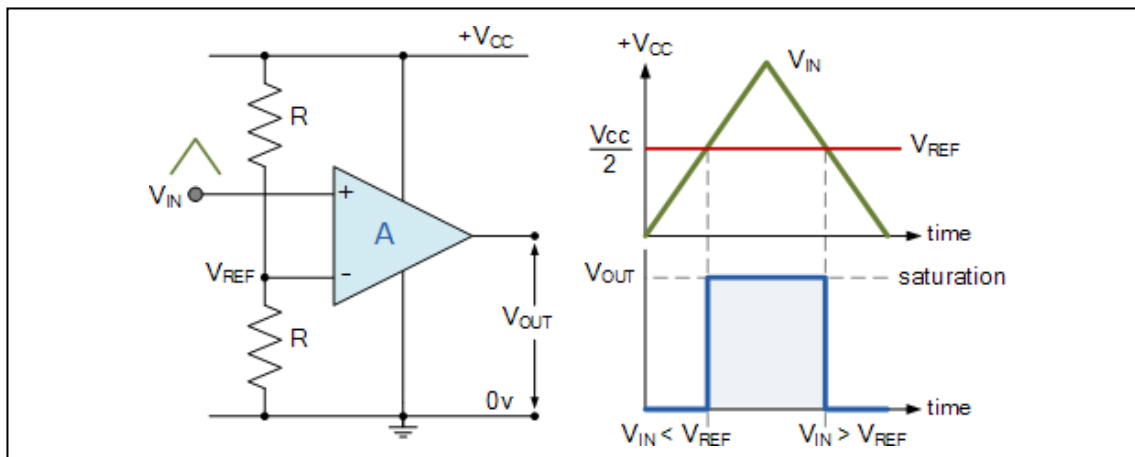
A comparator is a circuit used to sense when a varying signal reaches some threshold value. The simplest comparator circuit has the signal voltage directly to one of the input terminals and a reference voltage to the other. When the OP-AMP is used open loop, its output makes a transition between saturated states as the input signal passes through.

A basic OP-AMP comparator circuit can be used to detect either a positive or a negative going input voltage depending upon which input of the operational amplifier we connect the fixed reference voltage source and the input voltage to.

8.1. Negative voltage Comparator (inverting comparator) circuit detects when the input signal  $V_{in}$  is BELOW or more negative than the reference voltage. Then the comparator circuit will produce  $V_{out}$  which is HIGH as shown.

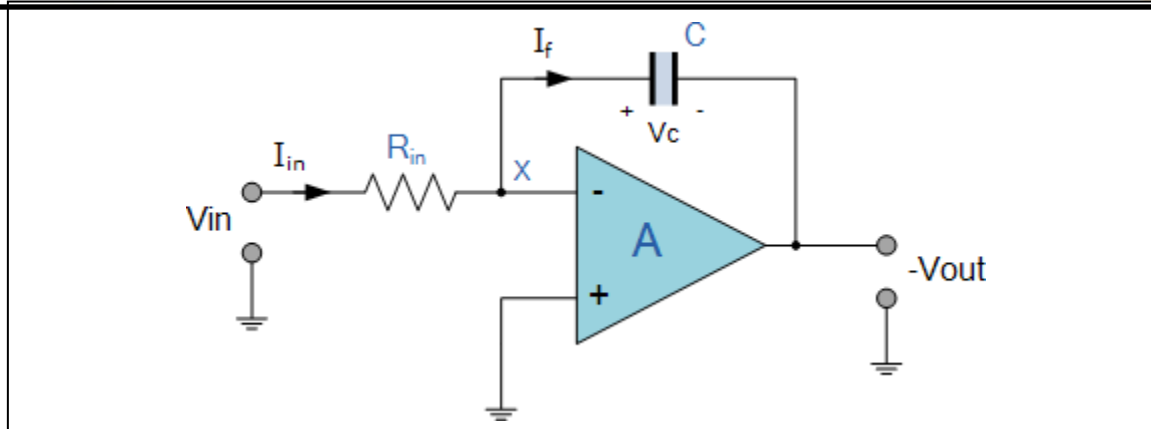


8.2. Positive Voltage Comparator (non-inverting) circuit detects when the input signal  $V_{in}$  is ABOVE or more positive than the reference voltage then the circuit produces an output at  $V_{out}$  which is HIGH as shown.



## 9. The OP-AMP as Integrator Circuit

As its name implies, the OP-AMP integrator is an operational amplifier circuit that performs the mathematical operational of integration. Figure below shows the OP-AMP integrator circuit.



According to the above circuit the  $V_{out}$  is:

$$I_1 = \frac{V_{in}}{R_{in}}$$

$$I_2 = -C \frac{dV_{out}}{dt}$$

(According to Capacitor current law)

$$\text{Since: } I_1 = I_2$$

$$\frac{V_{in}}{R} = -C \frac{dV_{out}}{dt}$$

$$-RC dV_{out} = V_{in} dt$$

$$dV_{out} = \frac{1}{-RC} V_{in} dt$$

$$\int_0^t dv_{out} = \frac{1}{-RC} \int_0^t V_{in} dt$$

$$V_{out(t)} - V_{out(0)} = \frac{1}{-RC} \int_0^t V_{in} dt$$

( assuming  $V_{out(0)} = 0$  )

$$V_{out(t)} = \frac{1}{-RC} \int_0^t V_{in} dt$$

Capacitor current law:

$$C = \frac{q}{v}$$

$$q = c v$$

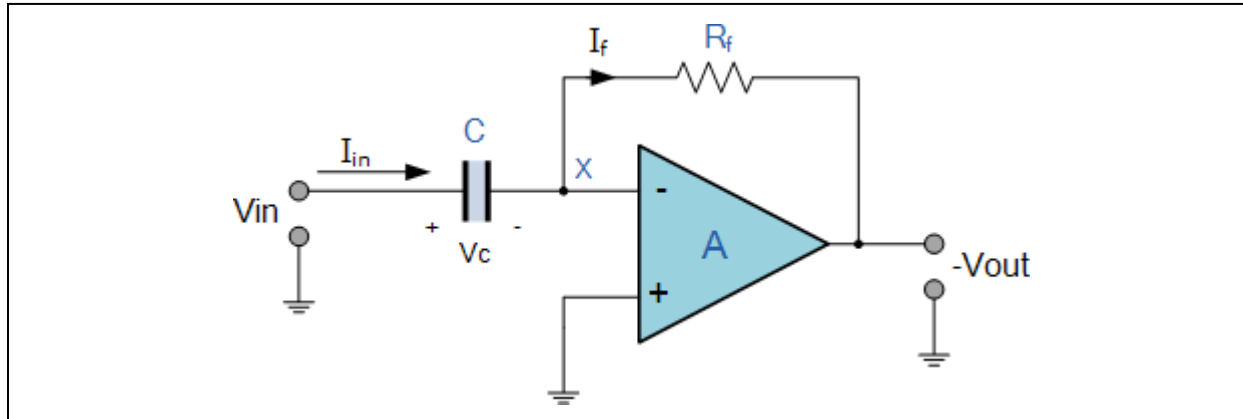
$$\frac{dq}{dt} = C \frac{dv}{dt}$$

$$I = C \frac{dv}{dt}$$



## 10. The OP-AMP as Differentiator Circuit:

The basic operational amplifier differentiator circuit produces an output signal which is the first derivative of the input signal. Figure below depicts the OP-AMP amplifier circuit.



According to the above circuit  $V_{out}$  is equal to:

$$I_{in} = I_f$$

$$C \frac{dv_c}{dt} = \frac{-V_{out}}{R_f}$$

$$\text{When } V_c = V_{in}$$

$$C \frac{dv_{in}}{dt} = \frac{-V_{out}}{R_f}$$

$$-V_{out} dt = C dv_{in} R_f$$

$$V_{out} = -C R_f \frac{dv_{in}}{dt}$$