Centroid and Center of Gravity:

The centroid is a point which defines the geometric center of an object. The lines, areas, and volumes all have centroids. We will study the centroids of plane , curve ,areas ,volume and composite bodies.

Centroid of a line in a plane:

The centroid C represents the center of a homogenous wire of length L and is specified by the distances $\bar{x}$ &$\bar{y}$, where:

$\bar{x}$: horizontal distance from the centroid to the y-axis,

$\bar{y}$: vertical distance from the centroid to the x-axis.

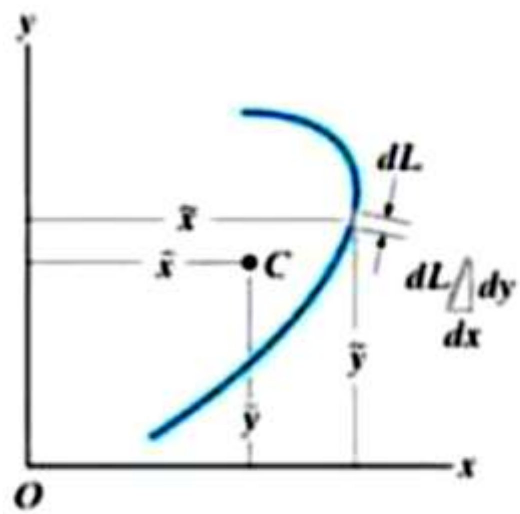
If the length L is subdivided into differential elements dl, then the moments of these elements about an axis is equal to the moment of total length about the same axis

$L.\ \bar{x} = \Sigma\ \tilde{x}\ .dl$

$\bar{x} = \Sigma\ \tilde{x}.dl\ /L$

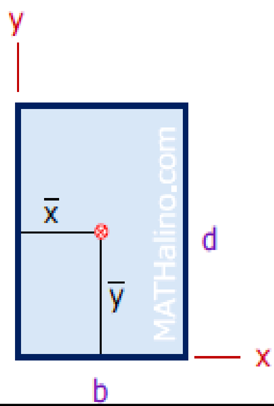
$L.\ \bar{y} = \Sigma\ \tilde{y}.dl$

$\bar{y} = \Sigma\ \tilde{y}.dl/L$



In integral form : $\bar{x} = \frac{\int \tilde{x}.dl}{L}$, $\bar{y} = \frac{\int \tilde{y}.dl}{L}$

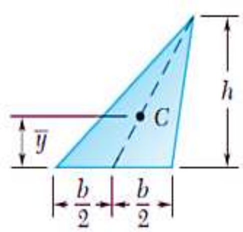
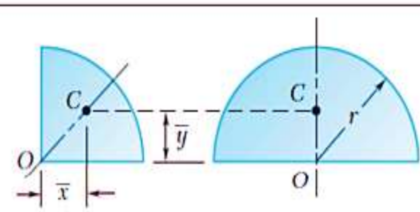
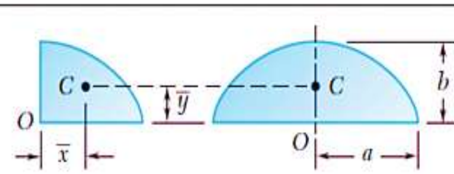
Rectangle — Area and Centroid

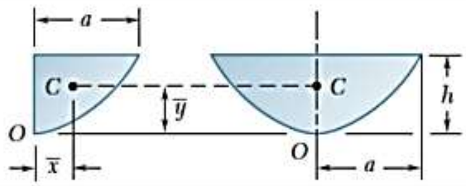
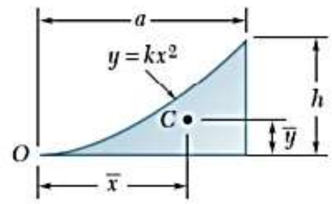
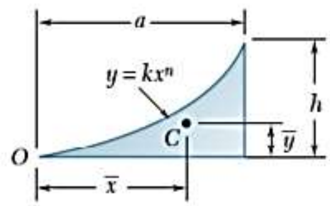
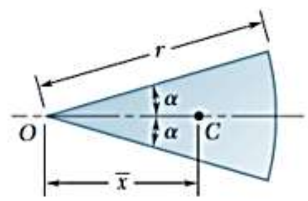


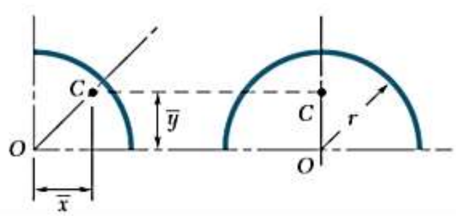
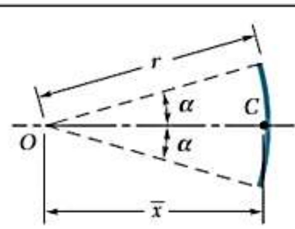
$$A = bd$$

$$\bar{x} = \frac{1}{2}b$$

$$\bar{y} = \frac{1}{2}d$$

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2

Shape		$\overline{x}$	$\overline{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

Centroid of a Volume:

$$\bar{x} = \frac{\int_V \tilde{x}\, dV}{\int_V dV} \qquad \bar{y} = \frac{\int_V \tilde{y}\, dV}{\int_V dV} \qquad \bar{z} = \frac{\int_V \tilde{z}\, dV}{\int_V dV}$$

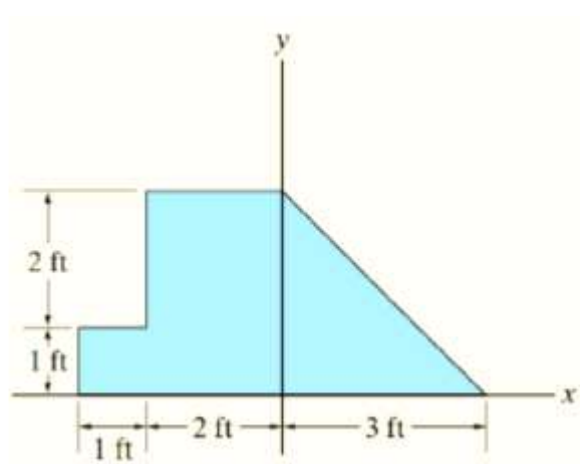
Centroid of composite areas:

The centroid of composite areas can be found using the relations :

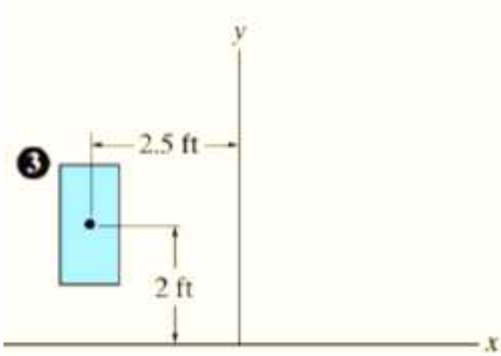
$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} \qquad \bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

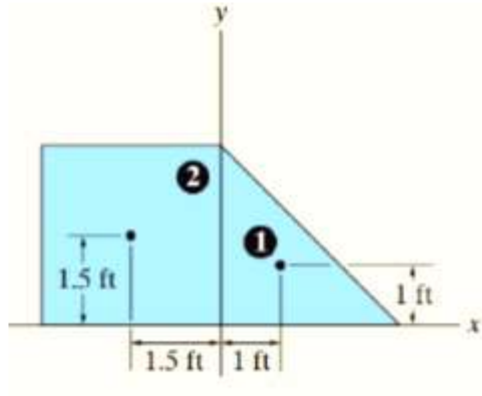
Where:
x ,y: centroids of each composite part of the area.
ƸA: sum of the areas of all parts (total areas).
$\bar{x}$,$\bar{y}$: centroids of the total area.

EXAMPLE(1): Locate the centroid of the plate area shown in figure below:



Sol:





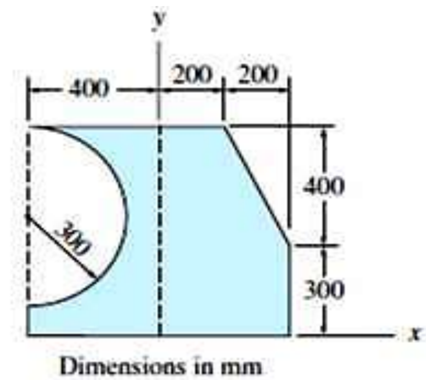
Segment	A (ft^2)	$\tilde{x}$ (ft)	$\tilde{y}$ (ft)	$\tilde{x}A$ (ft^3)	$\tilde{y}A$ (ft^3)
1	$\frac{1}{2}(3)(3) = 4.5$	1	1	4.5	4.5
2	$(3)(3) = 9$	-1.5	1.5	-13.5	13.5
3	$-(2)(1) = -2$	-2.5	2	5	-4
	$\Sigma A = 11.5$			$\Sigma \tilde{x}A = -4$	$\Sigma \tilde{y}A = 14$

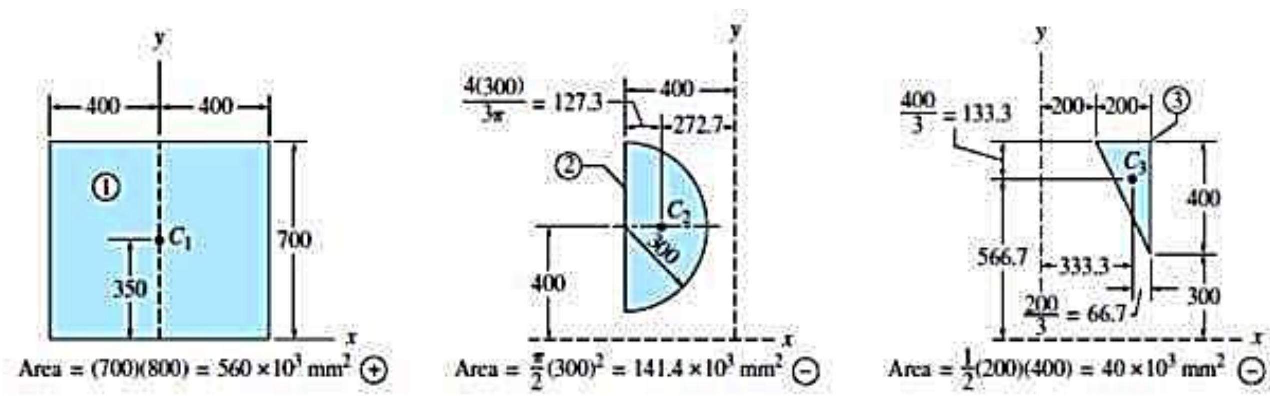
Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{-4}{11.5} = -0.348 \text{ ft} \qquad \textit{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{14}{11.5} = 1.22 \text{ ft} \qquad \textit{Ans.}$$

Ex.2: Using the method of composite areas, determine the location of the centroid of the shaded area shown in figure below.



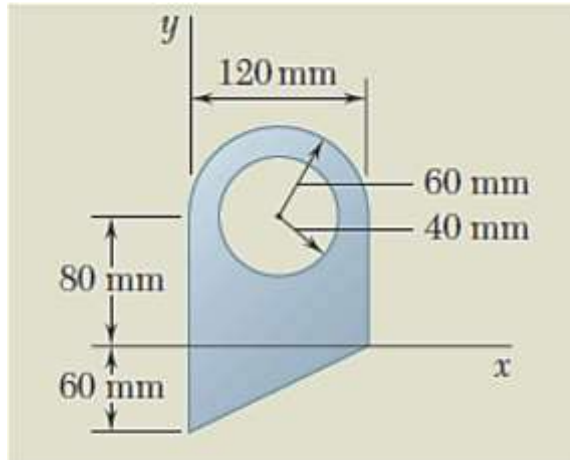


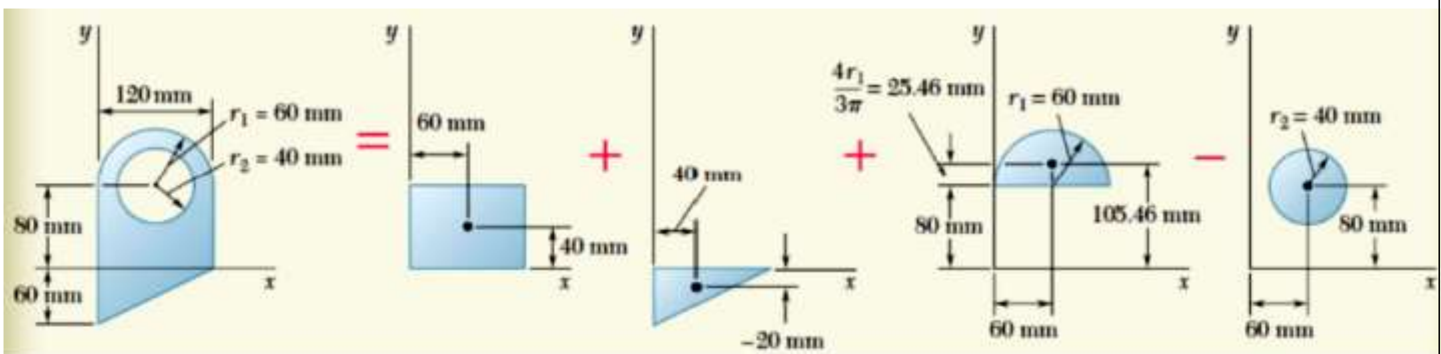
Shape	Area A (mm²)	$\bar{x}$ (mm)	$A\bar{x}$ (mm³)	$\bar{y}$ (mm)	$A\bar{y}$ (mm³)
1 (Rectangle)	$+560.0 \times 10^3$	0	0	+350	196.0×10^6
2 (Semicircle)	-141.4×10^3	−272.7	$+38.56 \times 10^6$	+400	-56.56×10^6
3 (Triangle)	-40.0×10^3	+333.3	-13.33×10^6	+566.7	-22.67×10^6
Σ	$+378.6 \times 10^3$	⋯	$+25.23 \times 10^6$	⋯	$+116.77 \times 10^6$

$$\bar{x} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{+25.23 \times 10^6}{+378.6 \times 10^3} = 66.6 \text{ mm}$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{+116.77 \times 10^6}{+378.6 \times 10^3} = 308 \text{ mm}$$

Example (3): For the plane area shown in Figure below find the location of the centroid.





Component	A, mm²	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm³	$\bar{y}A$, mm³
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A}$$

$= 757.7\times10^3/13.828\times10^3 = 54.8$ mm

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

$=506.2\times10^3/13.828\times10^3 = 36.6$ mm