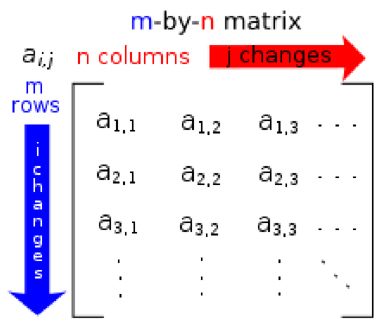
1.1 What is Matrices?

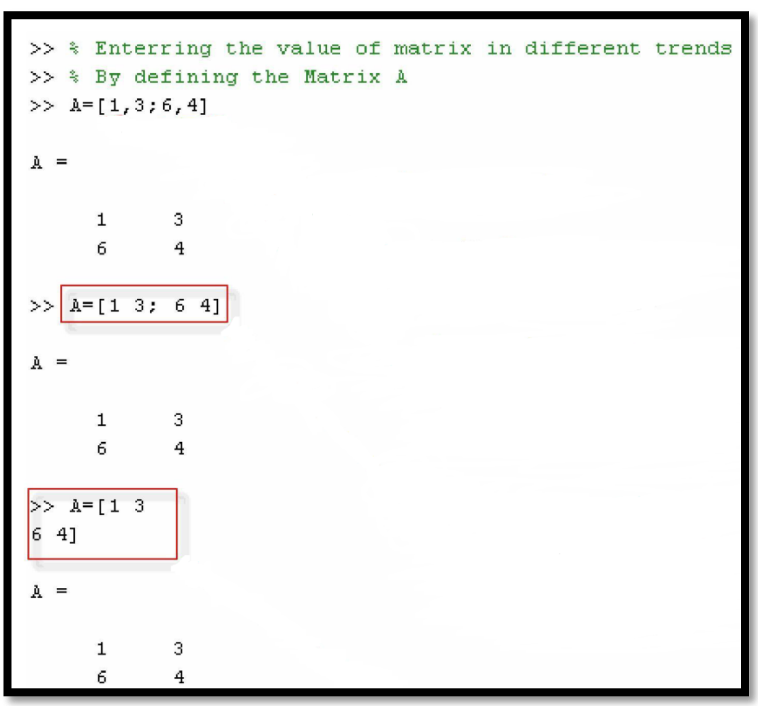
a matrix is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns and each item in a matrix are called its elements or entries.



For example, the dimensions of the matrix below are **2 × 3** (read "**two by three**"), because there are two rows and three columns.

$$\begin{bmatrix} 1 & 9 & -13 \\ 20 & 5 & -6 \end{bmatrix}$$

1.2 Entering Elements to Matrices



```
>> % Enterring the value of matrix in different trends
>> % By defining the Matrix A
>> A=[1,3;6,4]

A =

     1     3
     6     4

>> A=[1 3;  6 4]

A =

     1     3
     6     4

>> A=[1 3
6 4]

A =

     1     3
     6     4
```

1.3 Operations on Matrices

Once you are able to create a matrix, we can perform many standard operations on it such as summation, subtraction, multiplication, division and inverse of a matrix. All these operations can be made easily using Matlab program.

1.3.1 Matrices Summations (+)

Two matrices must have an equal number of rows and columns to be added. The sum of two matrices **A and B** will be a matrix which has the same number of rows and columns as do **A and B**. The sum of **A and B**, denoted **A + B**, is computed by adding corresponding elements of **A and B**.

$$\begin{bmatrix} 1 & 3 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 7 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 3+0 \\ 1+7 & 0+5 \\ 1+2 & 2+1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 8 & 5 \\ 3 & 3 \end{bmatrix}$$

1.3.2 Matrices Subtractions (-)

We can also subtract one matrix from another, as long as they have the same dimensions. **A − B** is computed by subtracting corresponding elements of **A and B**, and has the same dimensions as **A and B**.

$$\begin{bmatrix} 1 & 3 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 7 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1-0 & 3-0 \\ 1-7 & 0-5 \\ 1-2 & 2-1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -6 & -5 \\ -1 & 1 \end{bmatrix}$$

Next we will use matlab program in matrices subtractions and notice the resultus obtained from it.

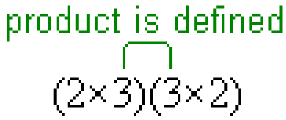
```
Command Window
>> % By Defining the Matrix A
>> A=[1 2;4 6;9 8];
>> % By Defining the Matrix B
>> B=[0 4;3 9;3 7];
>> % C=A-B
>> C=A-B

C =

     1    -2
     1    -3
     6     1
```

1.3.3 Matrices Multiplication (*)

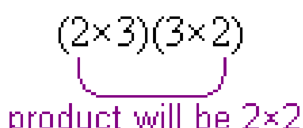
In order to multiply two matrices, **A and B**, the number of columns in **A** must equal the number of rows in **B**.

$$\mathbf{AB} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} 0 & 3 \\ -2 & -1 \\ 0 & 4 \end{bmatrix}$$

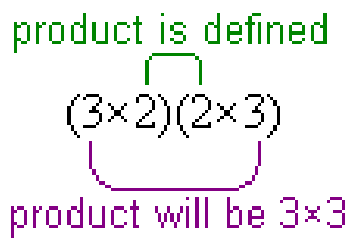
- ✓ You can use this fact to check quickly whether a given multiplication is defined. Write the product in terms of the matrix dimensions. In the case of the above problem, **A is 2×3** and **B is 3×2**, so **AB is(2×3)(3×2)**. The middle values match:

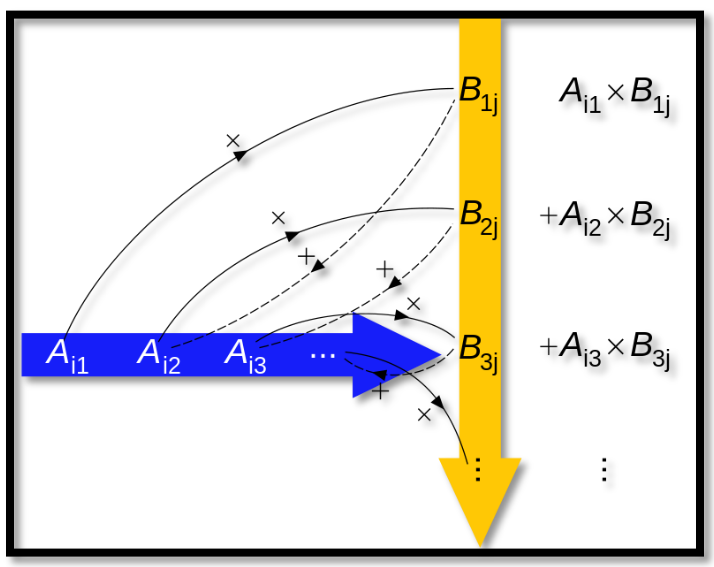


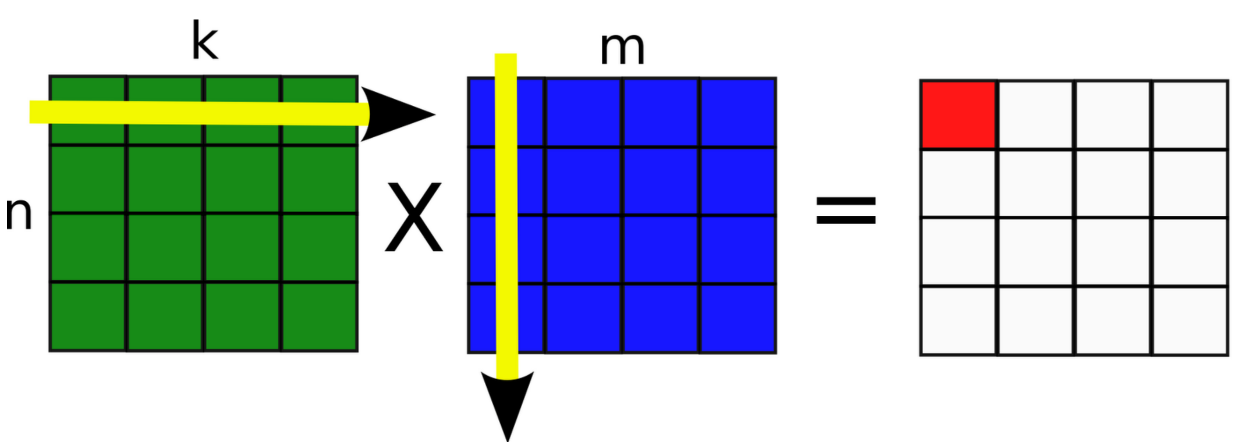
- ✓ The multiplication is defined, the product matrix 2×2. You can also see this on the dimensions



- ✓ Using this, you can see that **BA** must be a different matrix from **AB**, because:







Next we will use matlab program in matrices multiplications and notice the resultus obtained from it.

```
Command Window
>> % By defining the Matrix A
>> A=[1 2;4 6;9 8];
>> % By Defining the Matrix B
>> B=[0 3 3;4 9 7];
>> % C=A*B
>> C=A*B
```

$$C = \begin{matrix} 8 & 21 & 17 \\ 24 & 66 & 54 \\ 32 & 99 & 83 \end{matrix}$$

1.3.4 Matrices Division and Inverse

For matrices, there is no such thing as **division**. You can add, subtract, and multiply matrices, but you **cannot** divide them. There is a related concept, though, which is called **"inversion"**.

***The inverse of a 2 × 2 matrix**

In the case of a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ a simple formula exists to find its inverse:

$$\text{if} \quad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \text{then} \quad A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Example

Find the inverse of the matrix $A = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$.

Solution

Using the formula

$$A^{-1} = \frac{1}{(3)(2) - (1)(4)}\begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & -1 \\ -4 & 3 \end{pmatrix}$$

This could be written as

$$\begin{pmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{pmatrix}$$

***The inverse of a 3 × 3 matrix**

Before you work through this leaflet, you will need to know how to find the **determinant** and **cofactors** of a 3 × 3 matrix.

$$A = \begin{pmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{pmatrix} \qquad C = \begin{pmatrix} -2 & 3 & 9 \\ 8 & -11 & -34 \\ -5 & 7 & 21 \end{pmatrix}$$

In order to find the inverse of A, we first need to use the matrix of cofactors, C, to create the adjoint of matrix A. The adjoint of A, denoted adj(A), is the transpose of the matrix of cofactors:

$$\text{adj}(A) = C^T$$

Remember that to find the transpose, the rows and columns are interchanged

$$\text{adj}(A) = C^T = \begin{pmatrix} -2 & 8 & -5 \\ 3 & -11 & 7 \\ 9 & -34 & 21 \end{pmatrix}$$

Then the formula for the inverse matrix is

Given a matrix A, its inverse is given by

$$A^{-1} = \frac{1}{\det(A)}\text{adj}(A)$$

where $\det(A)$ is the determinant of A, and $\text{adj}(A)$ is the adjoint of A.

- **The inverse has the special property that**

$$A\,A^{-1} = A^{-1}\,A = I \qquad \text{(an identity matrix)}$$

Example

Find the inverse of $A = \begin{pmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{pmatrix}$.

Solution

We already have that $\text{adj}(A) = \begin{pmatrix} -2 & 8 & -5 \\ 3 & -11 & 7 \\ 9 & -34 & 21 \end{pmatrix}$.

In an earlier leaflet, the determinant of this matrix A was found to be 1. So

$$A^{-1} = \frac{1}{\det(A)}\text{adj}(A) = \frac{1}{1}\begin{pmatrix} -2 & 8 & -5 \\ 3 & -11 & 7 \\ 9 & -34 & 21 \end{pmatrix} = \begin{pmatrix} -2 & 8 & -5 \\ 3 & -11 & 7 \\ 9 & -34 & 21 \end{pmatrix}$$

You should verify this is correct by showing that $A\,A^{-1} = A^{-1}\,A = I$, the 3×3 identity matrix.

Properties of Matrices

Property	Example
Commutativity of Addition	A + B = B + A
Associativity of Addition	A + (B + C) = (A + B) + C
Associativity of Scalar Multiplication	(cd) A = c (dA)
Scalar Identity	1A = A(1) = A
Distributive	C (A + B) = CA + CB
Associativity of Multiplication	A (BC) = (AB) C
Multiplicative Identity	IA = AI = A