Al-Mustaqbal University College
Medical Physics Department
2nd Stage
Differential Equations
[redacted]

كلية المستقبل الجامعة
قسم الفيزياء الطبية
المرحلة الثانية
المعادلات التفاضلية
[redacted]

Bernoulli Equation

1. Theory

A **Bernoulli differential equation** can be written in the following standard form:

$$\boxed{\frac{dy}{dx} + P(x)y = Q(x)y^n} \quad ,$$

where $n \neq 1$ (the equation is thus **nonlinear**).

To find the solution, change the dependent variable from y to z, where $z = y^{1-n}$. This gives a differential equation in x and z that is **linear**, and can be solved using the integrating factor method.

Note: Dividing the above standard form by y^n gives:

$$\begin{aligned} \frac{1}{y^n}\frac{dy}{dx} + P(x)y^{1-n} &= Q(x) \\ \text{i.e. } \frac{1}{(1-n)}\frac{dz}{dx} + P(x)z &= Q(x) \end{aligned}$$

$\left(\text{where we have used } \frac{dz}{dx} = (1-n)y^{-n}\frac{dy}{dx}\right)$.

EXERCISE 1.
The general form of a Bernoulli equation is

$$\frac{dy}{dx} + P(x)y = Q(x)\,y^n ,$$

where P and Q are functions of x, and n is a constant. Show that the transformation to a new dependent variable $z = y^{1-n}$ reduces the equation to one that is linear in z (and hence solvable using the integrating factor method).

مدرس المادة

غفران هادي سعود

Bernoulli Differential Equation: lec.4

$$\boxed{y' + P(x)\cdot y = Q(x)\cdot y^n}$$ general form

* طريقة الحل: هي ان نقوم بعدد من الخطوات لتحويلها الى معادلة تفاضلية خطية . (linear eq.)

* خطوات الحل:

١- نقسم طرفي المعادلة على (y^n)

① $$\frac{y'}{y^n} + P(x)\cdot\frac{1}{y^{n-1}} = Q(x) \quad \text{-----} \; ①$$

٢- نفرض ان ($Z = \frac{1}{y^{n-1}}$)

② let $$Z = \frac{1}{y^{n-1}} \Rightarrow Z = y^{1-n} \quad \text{----} \; ②$$

"دائماً"

$Z = y^{1-n}$ لجميع الاسئلة

٣- نشتق الـ (Z) بالنسبة للـ (y)

③ $$Z' = (1-n)\cdot y^{-n}\cdot y' \quad \text{-----} \; ③$$

نعوض ② و ③ في ①

$$\frac{1}{n-1}\cdot\bar{Z} + P(X)\cdot Z = Q(X) \quad \text{(linear equation)}$$

EX1: solve $\left[\bar{y} + 2xy = -xy^4\right]$

Sol:

$$\left.\bar{y} + 2xy = -xy^4\right] \div y^4$$

$$\frac{\bar{y}}{y^4} + \frac{2x}{y^3} = -x \quad ---- \quad (1)$$

let $$\frac{1}{y^3} = Z \Longrightarrow Z = y^{-3} \quad ---- \quad (2)$$

$$\bar{Z} = -3y^{-4}\cdot\frac{dy}{dx}$$

$$\bar{Z} = -3y^{-4}\cdot\bar{y} \longrightarrow \bar{Z} = -3\frac{\bar{y}}{y^4} \quad ---- \quad (3)$$

نضرب المعادلة $(-3) * (1)$

$$-3\frac{\bar{y}}{y^4} - \frac{6x}{y^3} = 3x \quad ---- \quad (4)$$

sub 2 & 3 in (4)

$$\bar{Z} - 6xZ = 3x \quad \text{(linear D.E.)}$$

$P(x) = -6x \quad , \quad Q(x) = 3x$

$$IF = e^{\int p(x)\,dx} = e^{\int -6x\,dx} = e^{-3x^2}$$

$$y \cdot IF = \int [Q(x) + IF\,dx] + c$$

$$y \cdot e^{-3x^2} = \int (3x \cdot e^{-3x^2}\,dx) + c$$

$$y e^{-3x^2} = -\frac{1}{2}\int (-2 \cdot 3x \cdot e^{-3x^2}\,dx) + c$$

$$y e^{-3x^2} = -\frac{1}{2} e^{-3x^2} + c$$

Ex2: Solve $y' + 2xy = xy^3$

Sol:

$$[y' + 2xy = xy^3] \div y^3 \rightarrow n = 3$$

$$\frac{y'}{y^3} + \frac{2x}{y^2} = x \quad ---- \quad (1)$$

let $Z = \frac{1}{y^2}$ or $Z = y^{1-n} \rightarrow Z = y^{-2} \quad ---- \quad (2)$

$$Z' = -2y^{-3} \cdot y' \rightarrow Z' = \frac{-2y'}{y^3} \quad ---- \quad (3)$$

نضرب معادلة (1) * (-2) ونعتبرها معادلة (4)

$$-2\frac{y'}{y^3} - \frac{4x}{y^2} = -2x \quad ---- \quad (4)$$

نعوض 2 و 3 في 4

$\bar{Z} - 4xZ = -2x$ (linear D.E.)

$P(x) = -4x$

$Q(x) = -2x$

$IF = e^{\int -4x\,dx} = e^{-2x^2}$

$\therefore\ y\,IF = \int (Q(x) \cdot IF\,dx) + c$

$y \cdot e^{-2x^2} = \int (-2x \cdot e^{-2x^2}) + c$

$y e^{-2x^2} = \frac{1}{2} \int (2 \cdot -2x \cdot e^{-2x^2}) + c$

$y e^{-2x^2} = \frac{1}{2} \cdot e^{-2x^2} + c$